CHAPTER 7

Factoring Polynomials

Section 7A

Factoring Methods

- 7-1 Factors and Greatest Common Factors
  - Algebra Lab Model Factoring
- 7-2 Factoring by GCF
  - Algebra Lab Model Factorization of Trinomials
- 7-3 Factoring $x^2 + bx + c$
- 7-4 Factoring $ax^2 + bx + c$
  - Technology Lab Use a Graph to Factor Polynomials

Section 7B

Applying Factoring Methods

- 7-5 Factoring Special Products
- Connecting Algebra to Number Theory Mental Math
- 7-6 Choosing a Factoring Method

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 7

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
7-1 Lesson	Algebra Lab 7-2 Lesson	7-2 Lesson Algebra Lab	Algebra Lab 7-3 Lesson	7-3 Lesson 7-4 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
7-4 Lesson	Technology Lab	Multi-Step Test Prep Ready to Go On?	7-5 Lesson	Connecting Algebra to Number Theory
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	
7-6 Lesson	Multi-Step Test Prep Ready to Go On?	Chapter Review	Chapter Test	

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 7

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
7-1 Lesson Algebra Lab 7-2 Lesson	7-2 Lesson Algebra Lab 7-3 Lesson	7-3 Lesson 7-4 Lesson	Technology Lab Multi-Step Test Prep Ready to Go On?	7-5 Lesson Connecting Algebra to Number Theory
DAY 6	**DAY 7**			
7-6 Lesson Multi-Step Test Prep Ready to Go On?	Chapter Review Chapter Test			

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 7

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

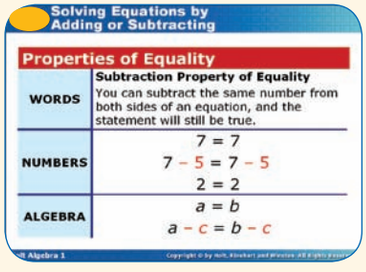
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

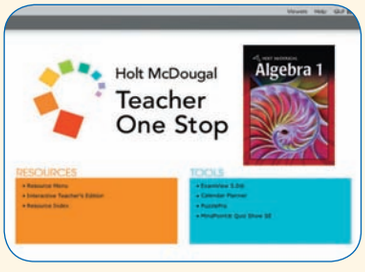
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 7

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Critical Thinking ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

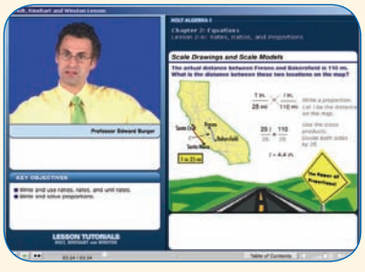
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

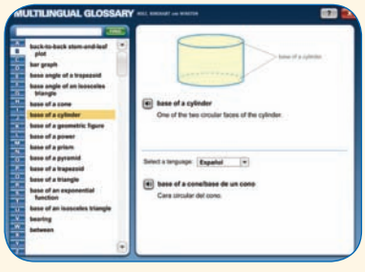
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



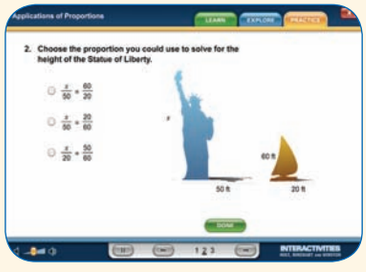
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 7

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!*® *Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!*® *Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView*™ *Assessment Suite

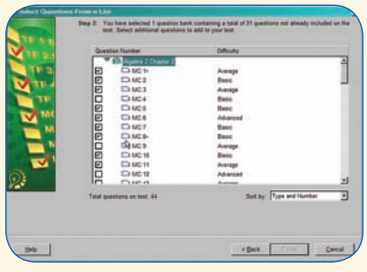
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView*™ *Assessment Suite
IDEA Works!*® *Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 7

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

B Chapter Test

Select the best answer.

1. Which is the prime factorization of 120?
   A $2 \cdot 2 \cdot 2 \cdot 15$ C $3 \cdot 5 \cdot 8$
   B $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ D $10 \cdot 12$
2. Find the GCF of 42 and 70.
   F 7 H 196
   G 14 J 210
3. Find the GCF of $30x^2$ and $45x^5$.
   A $5x^2$ C $15x^2$
   B $5x^5$ D $15x^5$
4. Kyle is making flower arrangements for a wedding. He has 16 roses and 60 carnations. Each arrangement will have the same number of flowers, but roses and carnations will not appear in the same arrangement. If he puts the greatest possible number of flowers in each arrangement, how many arrangements can he make?
   F 4 H 19
   G 15 J 38
5. Factor $30y^3 - 6y^2 + 12y$ completely.
   A $y(30y^2 - 6y + 12)$
   B $3y(10y^2 - 2y + 4)$
   C $6(5y^3 - y^2 + 2y)$
   D $6y(5y^2 - y + 2)$
6. Factor $2n(n + 3) - 5(n + 3)$.
   F $(n - 3)(2n + 5)$
   G $(n + 3)(2n + 5)$
   H $(n + 3)(2n - 5)$
   J cannot be factored
7. Factor $6a^3 - 3a^2 + 8a - 4$ by grouping.
   A $(2a - 1)(3a^2 + 4)$
   B $(2a + 4)(3a^2 - 1)$
   C $(6a^2 - 3a^2)(8a - 4)$
   D cannot be factored
8. Factor $x^2 + 8x + 12$.
   F $(x + 1)(x + 12)$
   G $(x + 2)(x + 6)$
   H $(x + 3)(x + 4)$
   J cannot be factored
9. Factor $x^2 - 3x + 70$.
   A $(x - 10)(x + 7)$
   B $(x - 7)(x - 10)$
   C $(x + 5)(x + 14)$
   D cannot be factored
10. Factor $x^2 - 6x - 16$.
    F $(x - 2)(x - 8)$
    G $(x - 2)(x + 8)$
    H $(x + 2)(x - 8)$
    J cannot be factored
11. Which value of b would make $x^2 + bx - 30$ factorable?
    A -31 C 11
    B -17 D 13
12. Write the factored form of the polynomial that is modeled by this geometric diagram.

$12x^2$	$9x$
$4x$	3

    F $(x + 3)(12x + 1)$
    G $(2x + 3)(6x + 1)$
    H $(3x + 1)(4x + 3)$
    J $(12x^2 + 4x)(9x + 3)$

B Chapter Test *(continued)*

13. Factor $5x^3 + 39x + 54$.
    A $(x + 2)(5x + 27)$
    B $(x + 3)(5x + 18)$
    C $(x + 6)(5x + 9)$
    D cannot be factored
14. Factor $8a^2 - 10a - 7$.
    F $(2a - 7)(4a + 1)$
    G $(2a - 1)(4a + 7)$
    H $(2a + 1)(4a - 7)$
    J cannot be factored
15. Which value of c would NOT make $3x^2 + 5x + c$ factorable?
    A -22 C 2
    B -2 D 22
16. Determine whether $n^2 - 10n - 25$ is a perfect square trinomial. If so, choose the correct factorization.
    F yes; $(n - 5)^2$
    G yes; $(n + 5)^2$
    H yes; $(n + 5)(n - 5)$
    J no
17. Determine whether $16x^2 + 24x + 9$ is a perfect square trinomial. If so, choose the correct factorization.
    A yes; $(4x - 3)^2$
    B yes; $(4x + 3)^2$
    C yes; $(4x + 3)(4x - 3)$
    D no
18. Determine whether $p^2 - 36$ is a difference of two squares. If so, choose the correct factorization.
    F yes; $(p - 6)^2$
    G yes; $(p + 6)(p - 6)$
    H yes; $(p + 18)(p - 18)$
    J no
19. Determine whether $4x^2 - 10$ is a difference of two squares. If so, choose the correct factorization.
    A yes; $(2x - 5)^2$
    B yes; $(2x + 5)(2x - 5)$
    C yes; $(2x + 10)(2x - 10)$
    D no
20. The area of a square is represented by $9x^2 - 12x + 4$. Which expression represents the perimeter of the square?
    F $3z - 2$ H $6z - 4$
    G $3z + 2$ J $12z - 8$
21. Is $x(4x^2 + 8x + 12)$ completely factored? If not, what other factoring can occur?
    A yes; the polynomial is completely factored.
    B no; 4 can be factored from each term of the trinomial.
    C no; the trinomial $4x^2 + 8x + 12$ can be factored into two binomials.
    D no; 4 can be factored from each term of the trinomial AND the resulting trinomial can be factored into two binomials.
22. Completely factor $3x^4 - 15x^2 - 18x^2$.
    F $x^2(3x + 2)(1x - 9)$
    G $3(x^2 + 1)(x^2 - 6)$
    H $3x^2(x + 1)(x - 6)$
    J cannot be factored
23. Completely factor $3m^3 + 5m^2 - 12m - 20$.
    A $(m^2 - 4)(3m + 5)$
    B $(m - 2)^2(3m + 5)$
    C $(m^2 + 4)(3m + 5)$
    D $(m + 2)(m - 2)(3m + 5)$

FREE RESPONSE

B Chapter Test

1. Write the prime factorization of 176.

Find the GCF.

2. 54 and 144
3. $30x^2$ and $66x^3$
4. Mrs. Mendoza is organizing seating for a standardized test. 45 ninth-grade students and 120 tenth-grade students will take the test. Each row will have the same number of students, but ninth-graders and tenth-graders will not be seated in the same row. If she puts the greatest possible number of students in each row, how many rows will there be?

Factor.

5. $14y^2 + 28y^2 - 54y$
6. $n(2n + 3) + 4(2n - 3)$
7. Factor $15a^3 + 20a^2 - 6a - 8$ by grouping.

Factor each trinomial.

8. $x^2 + 9x + 18$
9. $x^2 + 7x - 30$
10. $x^2 - 5x - 50$
11. Find an integer value of b that makes $x^2 + bx + 42$ factorable, and then factor the trinomial.

$b =$ _____

12. Write the polynomial modeled by this geometric diagram and then factor.

$18x^2$	$15x$
$6x$	5

B Chapter Test *(continued)*

Factor each trinomial.

13. $3x^2 + 47x + 140$
14. $27a^2 + 42a - 5$
15. Determine whether each value of c makes $5x^2 - 22x + c$ factorable. If so, factor it.

$c = -15$ _____

$c = 4$ _____

$c = 8$ _____

Determine whether the trinomial is a perfect square. If so, factor it. If not, explain why.

16. $n^2 + 72n + 36$
17. $9x^2 - 30x + 25$

Determine whether the binomial is a difference of two squares. If so, factor it. If not, explain why.

18. $p^2 - 132$
19. $16x^2 - 49$
20. The area of a square in square feet is represented by $4z^2 - 36z + 81$. Find an expression for the perimeter of the square. Then find the perimeter when $z = 10$ ft.

expression: _____

perimeter when $z = 10$ ft: _____

21. Tell whether $x(4x^2 + 19x + 12)$ is completely factored. If not, factor it.

Factor each polynomial completely.

22. $4x^5 - 30x^4 - 16x^2$
23. $2m^3 + 3m^2 - 18m - 27$

PERFORMANCE ASSESSMENT

Chapter Test

Scale Factors

Purpose

This performance task assesses the student's ability to factor by using the GCF, to factor trinomials including perfect square trinomials, and to find the scale factor of polygons when the areas are given as algebraic expressions.

Time

30 minutes

Grouping

Individuals

Preparation Hints

Review the formulas for the areas of rectangles and circles as needed. Review the fact that π is a number, and that it can be factored out of an expression like any other number.

Overview

Students are asked to factor expressions that represent the areas of similar rectangles and circles and identify the scale factor that was used. They are then asked to identify the relationship between the GCF of the polynomials and the scale factor.

Introduce the Task

Remind students that when every dimension of a figure is multiplied by the same number, the result is a similar figure (same shape, different size) and that the number that was the multiplier is called the scale factor. Tell students that they previously found scale factors as numeric values instead of algebraic expressions.

Performance Indicators

_____ Factors trinomials completely.
_____ Identifies the scale factor as the square of the ratio of the corresponding sides.
_____ Identifies the scale factor as the square root of the GCF.
_____ Evaluates polynomials.

Scoring Rubric

Level 4: Student solves all problems and gives good explanations.
Level 3: Student solves all problems, but does not give satisfactory explanations.
Level 2: Student solves only some problems, and attempts to give some explanations.
Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

Scale Factors

The area of rectangle A can be expressed by the polynomial $8x^2 + 2x - 3$. The area of rectangle B can be expressed by the polynomial $32x^2 + 8x - 12$.

1. Factor the expression for the area of rectangle A.
2. Factor the expression for the area of rectangle B.
3. Every dimension of rectangle A was multiplied by a scale factor to get rectangle B. What was the scale factor? Explain how you know.

The area of circle A can be expressed by the polynomial $4\pi x^2 - 20\pi x + 25\pi$. The area of circle B can be expressed by the polynomial $36\pi x^2 - 180\pi x + 225\pi$.

4. Factor the expression for the area of circle A. Then write an expression for the radius of circle A.
5. Factor the expression for the area of circle B.
6. What is the scale factor from circle A to circle B? How is it related to the GCF?
7. The area of parallelogram A is $3x^2 + 8x - 16$. The area of parallelogram B is $12x^2 + 32x - 64$. What is scale factor from parallelogram A to B?
8. The surface area of sphere A is $4\pi x^2 + 24\pi x + 36\pi$. The surface area of sphere B is $100\pi x^2 + 600\pi x + 900\pi$. What is the scale factor from sphere A to sphere B?
9. Evaluate the expressions in problem 8 for $x = 7$. (Leave your answers in terms of π.) Show how these values relate to your answer in problem 8.

CHAPTER 7

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

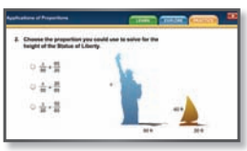
SECTION 7A
Factoring Methods

MULTI-STEP TEST PREP Students write and factor polynomials to model the distance traveled by an automobile.

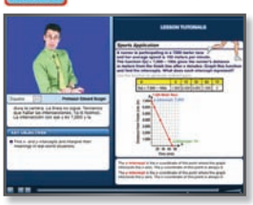
SECTION 7B
Applying Factoring Methods

MULTI-STEP TEST PREP Students factor polynomials to model the dimensions of a garden.

Interactivities Online ▶



Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 7

Factoring Polynomials

COMMON CORE

7A Factoring Methods

7-1 Factors and Greatest Common Factors
Lab Model Factoring A.SSE.2
7-2 Factoring by GCF A.SSE.2
Lab Model Factorization of Trinomials A.SSE.2
7-3 Factoring $x^2 + bx + c$ A.SSE.2
7-4 Factoring $ax^2 + bx + c$ A.SSE.2
Lab Use a Graph to Factor Polynomials A.SSE.3a

7B Applying Factoring Methods

7-5 Factoring Special Products A.SSE.2
7-6 Choosing a Factoring Method A.SSE.2

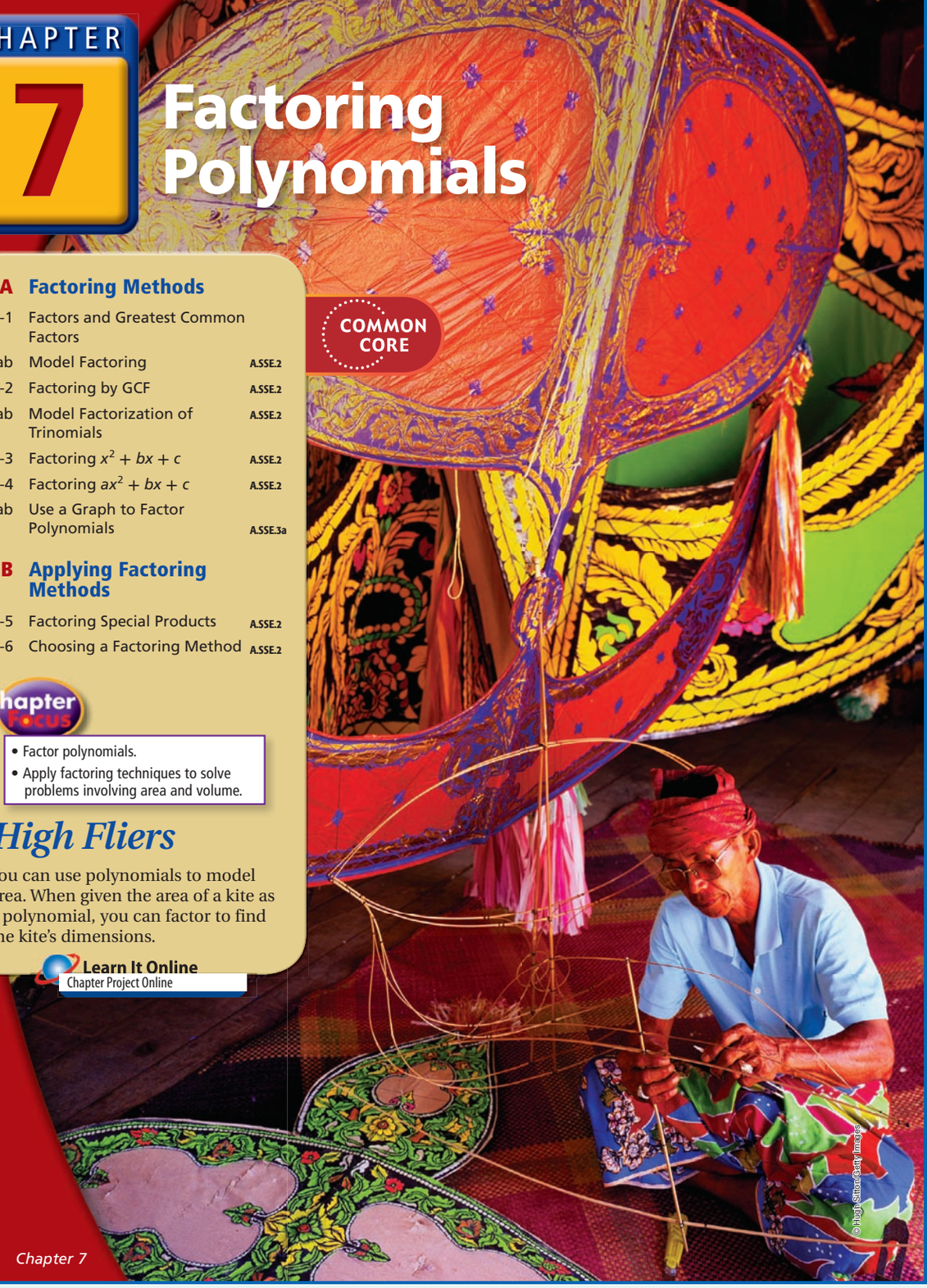
Chapter Focus

- Factor polynomials.
- Apply factoring techniques to solve problems involving area and volume.

High Fliers

You can use polynomials to model area. When given the area of a kite as a polynomial, you can factor to find the kite's dimensions.

Learn It Online
Chapter Project Online



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452 *Chapter 7*

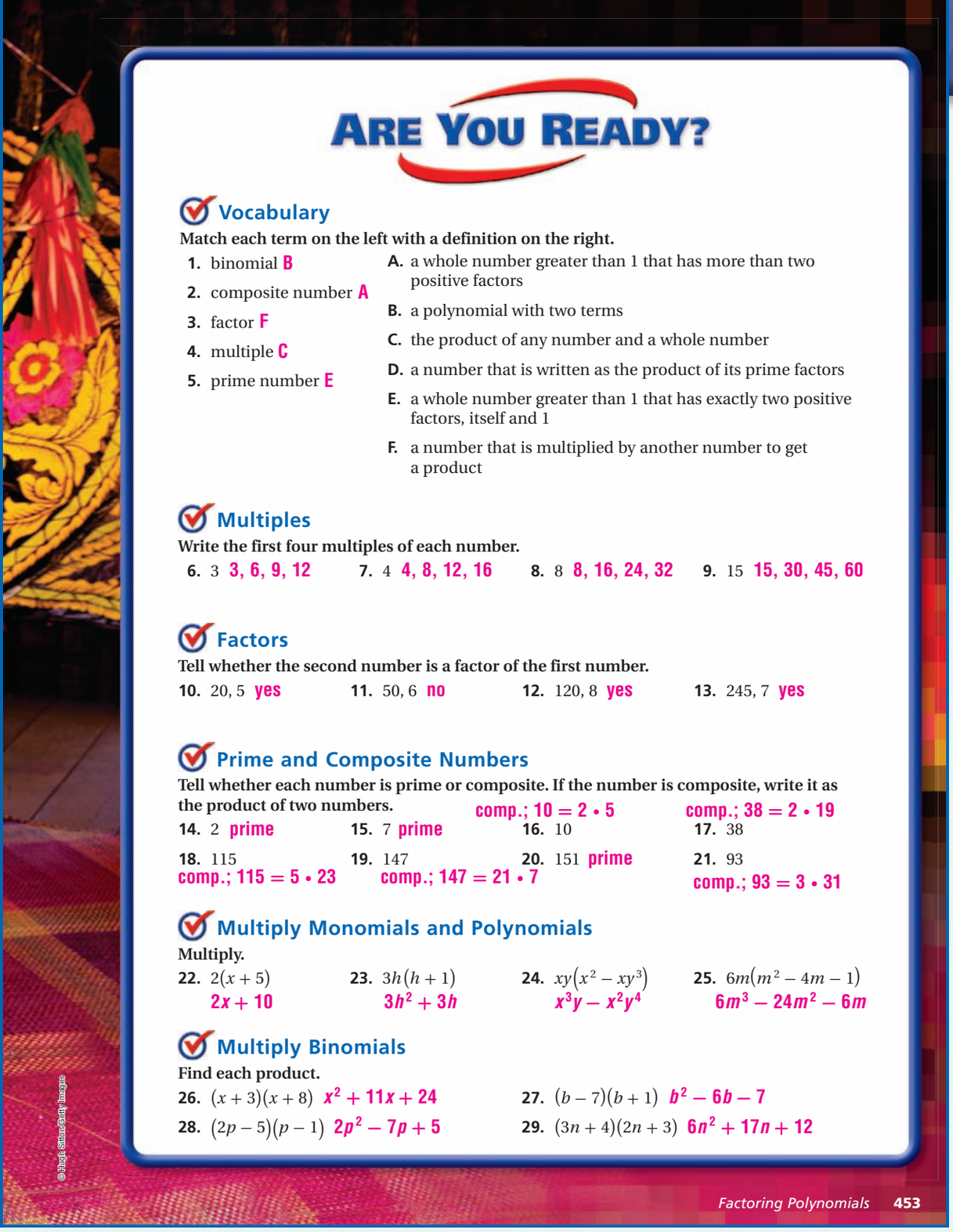
Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. binomial **B**
2. composite number **A**
3. factor **F**
4. multiple **C**
5. prime number **E**

A. a whole number greater than 1 that has more than two positive factors

B. a polynomial with two terms

C. the product of any number and a whole number

D. a number that is written as the product of its prime factors

E. a whole number greater than 1 that has exactly two positive factors, itself and 1

F. a number that is multiplied by another number to get a product

Multiples

Write the first four multiples of each number.

6. 3 **3, 6, 9, 12** **7.** 4 **4, 8, 12, 16** **8.** 8 **8, 16, 24, 32** **9.** 15 **15, 30, 45, 60**

Factors

Tell whether the second number is a factor of the first number.

10. 20, 5 **yes** **11.** 50, 6 **no** **12.** 120, 8 **yes** **13.** 245, 7 **yes**

Prime and Composite Numbers

Tell whether each number is prime or composite. If the number is composite, write it as the product of two numbers.

14. 2 **prime** **15.** 7 **prime** **16.** 10 **comp.; $10 = 2 \cdot 5$** **17.** 38 **comp.; $38 = 2 \cdot 19$**

18. 115 **comp.; $115 = 5 \cdot 23$** **19.** 147 **comp.; $147 = 21 \cdot 7$** **20.** 151 **prime** **21.** 93 **comp.; $93 = 3 \cdot 31$**

Multiply Monomials and Polynomials

Multiply.

22. $2(x + 5)$ **$2x + 10$** **23.** $3h(h + 1)$ **$3h^2 + 3h$** **24.** $xy(x^2 - xy^3)$ **$x^3y - x^2y^4$** **25.** $6m(m^2 - 4m - 1)$ **$6m^3 - 24m^2 - 6m$**

Multiply Binomials

Find each product.

26. $(x + 3)(x + 8)$ **$x^2 + 11x + 24$** **27.** $(b - 7)(b + 1)$ **$b^2 - 6b - 7$**

28. $(2p - 5)(p - 1)$ **$2p^2 - 7p + 5$** **29.** $(3n + 4)(2n + 3)$ **$6n^2 + 17n + 12$**

© Jhpl. Sitton/Getty Images

CHAPTER 7

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

- ***Are You Ready? Intervention and Enrichment* Worksheets**
- ***Are You Ready?* Online**

ARE YOU READY?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Prerequisite Skill	*Are You Ready? Intervention*, Chapter 7		
	Worksheets	Online	
Multiples	Skill 1	Activity 1	Diagnose and Prescribe Online
Factors	Skill 3	Activity 3	
Prime and Composite Numbers	Skill 5	Activity 5	
Multiply Monomials and Polynomials	Skill 62	Activity 62	
Multiply Binomials	Skill 64	Activity 64	

***Are You Ready? Enrichment*, Chapter 7**
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Answers to *Vocabulary Connections*

1. to write a number or polynomial as a multiplication of factors
2. Possible answer: Civilize/civilization, capitalize/capitalization, optimize/optimization, visualize/visualization; the suffix *-ization* indicates the result of an action that has taken place. Factorization is the result when something is factored.
3. The main part; the first coat of paint; fundamental; early stage of development; a prime factor is a basic building block of the product. It cannot be further factored.
4. A prime number has no positive factors other than 1 and itself. A prime factorization is written with factors that are all prime numbers.
5. *Common* means "alike." The greatest common factor of 2 numbers is the greatest factor that the 2 numbers share.

CHAPTER 7 Study Guide: Preview

Where You've Been

Previously, you

- used properties of exponents to evaluate and simplify expressions.
- added and subtracted polynomials by combining like terms.
- multiplied polynomials.

In This Chapter

You will study

- greatest common factors.
- how to factor polynomials.
- how to factor special products.
- how to choose a factoring method.

Where You're Going

You can use the skills in this chapter

- in Geometry to solve area problems.
- in Physics to solve quadratic equations.
- in the real world to calculate dimensions in landscaping, construction, or design work.

Key Vocabulary/Vocabulario

greatest common factor	máximo común divisor
prime factorization	factorización prima

Vocabulary Connections

To become familiar with the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *factor* refers to a number or polynomial that is multiplied by another number or polynomial to form a product. What do you think the word *factor* means when it is used as a verb (action word)?
2. List some words that end with the suffixes *-ize* or *-ization*. What does the ending *-ization* seem to mean? What do you think *factorization* means?
3. The words *prime*, *primer*, *primary*, and *primitive* all come from the same root word. What are the meanings of these words? How can their meanings help you understand what a *prime factor* is?
4. What is a prime number? How might the **prime factorization** of a number differ from another factorization?
5. What does the word *common* mean? How can you use this meaning to understand the term **greatest common factor**?

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 459–461, 467–469, 477–478, 485, 488, 491, 494–495, 501–503, 504
2. **Reason abstractly and quantitatively.** Pages 461, 478–479, 486
3. **Construct viable arguments and critique the reasoning of others.** Pages 458, 459–460, 466, 468–469, 475, 476–479, 483, 485, 493, 494–495, 500, 501–502
4. **Model with mathematics.** Pages 477, 484–485
5. **Use appropriate tools strategically.** Pages 462, 470, 487, 498
6. **Attend to precision.** Pages 477, 502
7. **Look for and make use of structure.** Pages 460, 477–478, 485–486, 494–496
8. **Look for and express regularity in repeated reasoning.** Pages 490–491, 493, 502

Reading Strategy: Read a Lesson for Understanding

To help you learn new concepts, you should read each lesson with a purpose. As you read a lesson, make notes. Include the main ideas of the lesson and any questions you have. In class, listen for explanations of the vocabulary, clarification of the examples, and answers to your questions.

Reading Tips

Objectives
Evaluate and multiply by powers of 10.

Convert between standard notation and scientific notation.

The objectives tell you the main idea of the lesson.

If a power of 10 has a negative integer exponent, does that make the number negative?
How do I enter numbers written in scientific notation into my calculator?

Write down questions you have as you read the lesson.

EXAMPLE 1 Evaluating Powers of 10

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

0. 0 0 1

0.001

Work through the examples and write down any questions you have.

CHECK IT OUT!

Practice what you've learned in the Check It Out sections.

Try This

Read the lesson for your next class. Then answer the questions below.

1. What are the lesson objectives?
2. What vocabulary, formulas, and symbols are new?
3. Which examples, if any, are unclear?
4. What questions do you have about the lesson?

COMMON CORE **Standards for Mathematical Content** High School

SEEING STRUCTURE IN EXPRESSIONS Interpret the structure of expressions		
CC.9-12.A.SSE.2	Use the structure of an expression to identify ways to rewrite it.	Lab 7-2, 7-2, Lab 7-3, 7-3, 7-4, Lab 7-4, 7-5, 7-6
SEEING STRUCTURE IN EXPRESSIONS Write expressions in equivalent forms to solve problems		
CC.9-12.A.SSE.3a	Factor a quadratic expression to reveal the zeros of the function it defines.	Lab 7-4

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE **Online Edition**

Reading Strategy: Read a Lesson for Understanding

Discuss Discuss with students what options they have if there is not enough time during class to ask a question. Students may need to ask a classmate or talk to the teacher after class or after school.

Extend Before each lesson in this chapter, students should read the lesson and make notes similar to those shown on this page. Tell students to make sure that all their questions are answered before they start their homework.

Answers to *Try This*

1. Write the prime factorization of numbers. Find the GCF of monomials.
2. prime factorization, greatest common factor
3. Check students' work.
4. Check students' work.

Reading Connection

***Secrets of Mental Math* by Arthur Benjamin and Michael Shermer**

Quick: What's the fifth root of 11,881,376? Easy: 26! This book is a fascinating guide to rapid mental calculation, which has practical use as a check on calculator work.

Activity Ask students to choose and practice a mental math technique from the book. Have each student teach the technique to a partner, and learn his or her partner's technique.

SECTION 7A

Factoring Methods

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 7-1 Factors and Greatest Common Factors • Write the prime factorization of numbers. • Find the GCF of monomials. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☐ SAT Subject Tests		**Optional** colored counters (MK)
Algebra Lab Model Factoring • Use algebra tiles to model and factor polynomials. ☐ SAT-10 ☐ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK)
Lesson 7-2 Factoring by GCF • Factor polynomials by using the greatest common factor. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** algebra tiles (MK)
Algebra Lab Model Factorization of Trinomials • Use algebra tiles to model and factor trinomials. ☐ SAT-10 ☐ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	**Required** algebra tiles (MK) **Optional** transparency mat, transparency algebra tiles (MK)
Lesson 7-3 Factoring $x^2 + bx + c$ • Factor quadratic trinomials of the form $x^2 + bx + c$. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Lesson 7-4 Factoring $ax^2 + bx + c$ • Factor quadratic trinomials of the form $ax^2 + bx + c$. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** algebra tiles (MK)
Technology Lab Use a Graph to Factor Polynomials • Use a graphing calculator to factor a polynomial. ☐ SAT-10 ☑ NAEP ☐ ACT ☐ SAT ☐ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator

MK = *Manipulatives Kit*

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 39, 41, 43, 45, 47, 59, 51, 53, 55, 57, 61, 63, 65

Lesson 2: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 31, 33, 37, 39, 41, 43, 45, 47, 49, 51, 53, 57, 61, 63a, 63b, 63c, 67, 69a, 69b, 69c, 69d, 71, 73, 75, 77, 79

Lesson 3: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 59, 51, 55, 57, 59, 61a, 61b, 61c, 63, 65, 67, 69, 71a, 71b, 71c, 73, 75, 77 79, 81, 83, 85a, 85b, 85c, 85d, 85e

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 69a, 69b, 69c, 71, 73, 77, 79, 81, 83, 85, 87, 89

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

FACTORING NUMBERS

Lesson 7-1

Understanding factoring begins with an understanding of prime numbers. We define a *prime number* as a whole number greater than 1 that has exactly two whole numbers as factors, itself and 1. The first few prime numbers form the familiar sequence 2, 3, 5, 7, 11, ...

More than 2000 years ago, Euclid proved that there are infinitely many prime numbers. His proof went as follows: Suppose there were a finite number of primes. Then they could be listed in their entirety as $p_1, p_2, ..., p_n$. Consider the number that is the product of all the primes plus one; that is, $p_1 \cdot p_2 \cdot ... \cdot p_n + 1$. This number is not divisible by any of the primes in the list since division by any of these primes would result in a remainder of 1. This means that the number itself must be prime. However, it is greater than any of the primes in the list, so it cannot be one of the primes in the list. This contradicts the assumption that the list includes all of the primes. Thus, the original assumption that there is a finite number of primes must be false.

THE FUNDAMENTAL THEOREM OF ARITHMETIC

Lesson 7-1

The Fundamental Theorem of Arithmetic (also known as the Unique Factorization Theorem) states that every whole number greater than 1 is either a prime number or may be written as a unique product of prime numbers. For example, the number 495 may be written as the product $3 \cdot 3 \cdot 5 \cdot 11$ (or $3^2 \cdot 5 \cdot 11$), and the Fundamental Theorem of Arithmetic asserts that this is the *only* prime factorization, with the exception of different orderings of the factors. One result of this theorem is that when finding the prime factorization of the same number, two students may use very different methods, but in the end they will always arrive at the same prime factors.

The proof of the theorem has two parts: showing that every whole number greater than 1 may be written as a product of primes and showing that this factorization is unique.

The first part is straightforward. Suppose there were one or more whole numbers greater than 1 that could not be written as a product of prime numbers. Let k be the least of all such numbers. Then k cannot be prime, since every prime number is the product of a single prime (namely, itself). Thus, k must be composite and so $k = ab$ where a and b are whole numbers less than k. Since k was taken to be the least whole number that cannot be written as a product of primes, we know that a and b can be written as a product of primes: $a = p_1 \cdot p_2 \cdot ... \cdot p_m$ and $b = q_1 \cdot q_2 \cdot ... \cdot q_n$. So $k = p_1 \cdot p_2 \cdot ... \cdot p_m \cdot q_1 \cdot q_2 \cdot ... \cdot q_n$, which contradicts the initial assumption that k cannot be written as a product of prime numbers. Thus, every whole number greater than 1 must have a prime factorization.

The proof that the prime factorization is unique requires some basic number theory and will not be provided here, but it is worth noting that the standard proof is once again a proof by contradiction.

Students sometimes wonder why 1 is not considered prime. One reason is the fact that including 1 among the prime numbers would mean that prime factorizations are no longer unique. For example, we could write $495 = 1 \cdot 3^2 \cdot 5 \cdot 11$, $495 = 1^2 \cdot 3^2 \cdot 5 \cdot 11$, and so on.

However, it *is* possible to consider negative factors, and in fact, students must consider negative factors when factoring polynomials. From this perspective, a prime number p has exactly four factors (p, $-p$, 1, and -1) and integers such as -5 and -17 would be considered prime. The Fundamental Theorem of Arithmetic would no longer hold, however. For example, the number 18 could be written as a product of primes as $2 \cdot 3^2$ or $2 \cdot (-3)^2$.

Finally, prime factorization gives a convenient method of identifying *all* whole number factors of a number. For example, any factor of 495 must have the form $3^k \cdot 5^m \cdot 11^n$, where $k = 0$, 1, or 2, $m = 0$ or 1, and $n = 0$ or 1. Taking the various combinations shows that the whole number factors of 495 are 1, 3, 5, 9, 11, 15, 33, 45, 55, 99, 165, and 495.

7-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Write the prime factorization of numbers.

Find the GCF of monomials.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Tell whether the second number is a factor of the first number.

1. 50, 6 no **2.** 105, 7 yes

3. List the factors of 28. ±1, ±2, ±4, ±7, ±14, ±28

Tell whether each number is prime or composite. If the number is composite, write it as the product of two numbers.

4. 11 prime

5. 98 composite; 49 • 2

Also available online

Math Humor

Q: Why couldn't 2 get along with the odd numbers?

A: They had nothing in common.

Learn It Online
State Resources Online

COMMON CORE

7-1 Factors and Greatest Common Factors

Objectives
Write the prime factorization of numbers.

Find the GCF of monomials.

Vocabulary
prime factorization
greatest common factor

Who uses this?
Web site designers who sell electronic greeting cards can use the greatest common factor of numbers to design their Web sites. (See Example 4.)

The numbers that are multiplied to find a product are called *factors* of that product. A number is divisible by its factors.

You can use the factors of a number to write the number as a product. The number 12 can be factored several ways.

Factorizations of 12

$1 \cdot 12$ $2 \cdot 6$ $3 \cdot 4$ $1 \cdot 4 \cdot 3$ $2 \cdot 2 \cdot 3$

Remember!
A prime number is a whole number that has exactly two positive factors, itself and 1. The number 1 is not prime because it only has one positive factor.

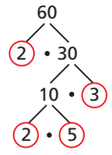
The order of the factors does not change the product, but there is only one example above that cannot be factored further. The circled factorization is the **prime factorization** because all the factors are prime numbers. The prime factors can be written in any order, and, except for changes in the order, there is only one way to write the prime factorization of a number.

EXAMPLE 1 Writing Prime Factorizations

Write the prime factorization of 60.

Method 1 Factor tree

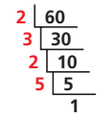
Choose any two factors of 60 to begin. Keep finding factors until each branch ends in a prime factor.



$60 = 2 \cdot 2 \cdot 5 \cdot 3$

Method 2 Ladder diagram

Choose a prime factor of 60 to begin. Keep dividing by prime factors until the quotient is 1.



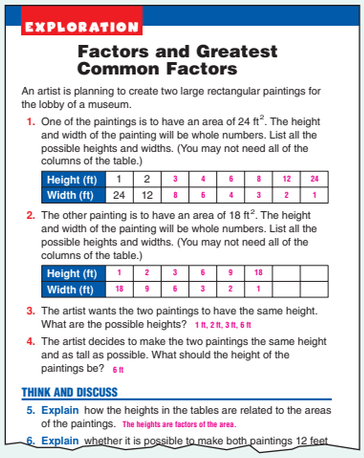
$60 = 2 \cdot 3 \cdot 2 \cdot 5$

The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

CHECK IT OUT! **Write the prime factorization of each number.**

1a. 40 $2^3 \cdot 5$ **1b.** 33 $3 \cdot 11$ **1c.** 49 7^2 **1d.** 19 19

1 Introduce


EXPLORATION

Factors and Greatest Common Factors

An artist is planning to create two large rectangular paintings for the lobby of a museum.

1. One of the paintings is to have an area of 24 ft^2. The height and width of the painting will be whole numbers. List all the possible heights and widths. (You may not need all of the columns of the table.)

Height (ft)	1	2	3	4	6	8	12	24
Width (ft)	24	12	8	6	4	3	2	1

2. The other painting is to have an area of 18 ft^2. The height and width of the painting will be whole numbers. List all the possible heights and widths. (You may not need all of the columns of the table.)

Height (ft)	1	2	3	6	9	18		
Width (ft)	18	9	6	3	2	1		

3. The artist wants the two paintings to have the same height. What are the possible heights? 1 ft, 2 ft, 3 ft, 6 ft
4. The artist decides to make the two paintings the same height and as tall as possible. What should the height of the paintings be? 6 ft

THINK AND DISCUSS

5. **Explain** how the heights in the tables are related to the areas of the paintings. The heights are factors of the area.
6. **Explain** whether it is possible to make both paintings 12 feet

Motivate

Ask students what it means for two people to have something in common (e.g., to have the same hair color, belong to the same club, etc.). Then ask students what it means for two numbers to have a common factor. What would the greatest common factor of two numbers be? Tell students that in this lesson, they will learn how to find the greatest common factor of numbers and monomials.

Explorations and answers are provided in your online resources.

Factors that are shared by two or more whole numbers are called *common factors*. The greatest of these common factors is called the **greatest common factor**, or GCF.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 32: 1, 2, 4, 8, 16, 32
Common factors: 1, 2, 4
The greatest of the common factors is 4.

EXAMPLE 2 Finding the GCF of Numbers

Find the GCF of each pair of numbers.

A **24 and 60**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — *List all the factors.*
factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 — *Circle the GCF.*

The GCF of 24 and 60 is 12.

B **18 and 27**

Method 2 Use prime factorization.

$18 = 2 \cdot 3 \cdot 3$ — *Write the prime factorization of each number.*
$27 = 3 \cdot 3 \cdot 3$ — *Align the common factors.*
$3 \cdot 3 = 9$

The GCF of 18 and 27 is 9.

CHECK IT OUT! **Find the GCF of each pair of numbers.**

2a. 12 and 16 **4** **2b.** 15 and 25 **5**

You can also find the GCF of monomials that include variables. To find the GCF of monomials, write the prime factorization of each coefficient and write all powers of variables as products. Then find the product of the common factors.

EXAMPLE 3 Finding the GCF of Monomials

Find the GCF of each pair of monomials.

A **$3x^3$ and $6x^2$**

$3x^3 = 3 \cdot x \cdot x \cdot x$ — *Write the prime factorization of each coefficient and write powers as products.*
$6x^2 = 2 \cdot 3 \cdot x \cdot x$ — *Align the common factors.*
$3 \cdot x \cdot x = 3x^2$ — *Find the product of the common factors.*

The GCF of $3x^3$ and $6x^2$ is $3x^2$.

B **$4x^2$ and $5y^3$**

$4x^2 = 2 \cdot 2 \cdot x \cdot x$ — *Write the prime factorization of each coefficient and write powers as products.*
$5y^3 = 5 \cdot y \cdot y \cdot y$ — *Align the common factors.*
There are no common factors other than 1.

The GCF of $4x^2$ and $5y^3$ is 1.

CHECK IT OUT! **Find the GCF of each pair of monomials.**

3a. $18g^2$ and $27g^3$ **$9g^2$** **3b.** $16a^6$ and $9b$ **1** **3c.** $8x$ and $7v^2$ **1**

Helpful Hint

If two terms contain the same variable raised to different powers, the GCF will contain that variable raised to the lower power.

COMMON ERROR ALERT

In **Example 1,** when using a ladder diagram to find the prime factorization of a number, students might include 1 as a prime factor. Remind students that 1 is not a prime number.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Write the prime factorization of 98. $2 \cdot 7^2$

Example 2

Find the GCF of each pair of numbers.

A. 100 and 60 20

B. 26 and 52 26

Example 3

Find the GCF of each pair of monomials.

A. $15x^3$ and $9x^2$ $3x^2$

B. $8x^2$ and $7y^3$ 1

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- Is the order of the prime factors important?
- How can you check your answer?

EXAMPLE 2

- How can you determine whether you have found all the factors of a number when using Method 1?
- Can the GCF of two numbers be one of the numbers? Explain.

EXAMPLE 3

- If two terms contain the same variable raised to different powers, what power of the variable will the GCF include? Explain.
- What happens if the monomials do not contain the same variable?

2 Teach

Guided Instruction

Review prime and composite numbers. Show students that different factor trees can be drawn for some numbers, but they all give the same prime factorization. Show that the GCF of two numbers or monomials is the largest term that divides into both of the numbers or monomials.

Reaching All Learners

Through Number Sense

Discuss with students how to quickly find the GCF of two numbers. If one number is a factor of the other number, it is the GCF of the two numbers. If not, then check the factors of the smaller number from largest to smallest until one is a factor of the greater number. This is the GCF of the two numbers.

Power Presentations with PowerPoint®

Additional Examples

Example 4

A cafeteria has 18 chocolate-milk cartons and 24 regular-milk cartons. The cook wants to arrange the cartons with the same number of cartons in each row. Chocolate and regular milk will not be in the same row. How many rows will there be if the cook puts the greatest possible number of cartons in each row? 7

Also available online

Questioning Strategies

EXAMPLE 4

- Is the GCF the answer to the problem? Explain.

Teaching Tip **Kinesthetic** Suggest that students use different colored counters to model the items in **Example 4.** These can be found in the Manipulatives Kit (MK).

EXAMPLE 4 ***Technology Application***

Garrison is creating a Web page that offers electronic greeting cards. He has 24 special occasion designs and 42 birthday designs. The cards will be displayed with the same number of designs in each row. Special occasion and birthday designs will not appear in the same row. How many rows will there be if Garrison puts the greatest possible number of designs in each row?

The 24 special occasion designs and 42 birthday designs must be divided into groups of equal size. The number of designs in each row must be a common factor of 24 and 42.

factors of 24: 1, 2, 3, 4, (6), 8, 12, 24
factors of 42: 1, 2, 3, (6), 7, 14, 21, 42

Find the common factors of 24 and 42.

The GCF of 24 and 42 is 6.

The greatest possible number of designs in each row is 6. Find the number of rows of each group of designs when there are 6 designs in each row.

$$\frac{24 \text{ special occasion designs}}{6 \text{ designs per row}} = 4 \text{ rows}$$

$$\frac{42 \text{ birthday designs}}{6 \text{ designs per row}} = 7 \text{ rows}$$

When the greatest possible number of designs is in each row, there are 11 rows in total.

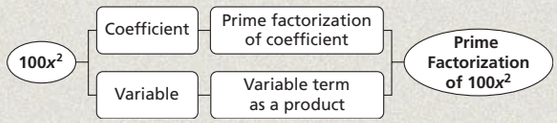
CHECK IT OUT! **4.** Adrianne is shopping for a CD storage unit. She has 36 CDs by pop music artists and 48 CDs by country music artists. She wants to put the same number of CDs on each shelf without putting pop music and country music CDs on the same shelf. If Adrianne puts the greatest possible number of CDs on each shelf, how many shelves does her storage unit need? 7

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Describe two ways you can find the prime factorization of a number.

Know it! Note **2. GET ORGANIZED** Copy and complete the graphic organizer. Show how to write the prime factorization of $100x^2$ by filling in each box.



3 Close

Summarize

Have students find the GCF of the following.

56 and 70 14
15 and 30 15
35 and 24 1
$36x^3$ and $50x$ $2x$
$10x^2$ and $25x^4y^2$ $5x^2$

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. Use a factor tree or divide the number by prime factors until the quotient is 1.
2. See Additional Answers.

7-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Define the term *greatest common factor* in your own words.
Possible answer: the greatest number that is a factor of two or more given numbers

SEE EXAMPLE 1

Write the prime factorization of each number.

2. 20 $2^2 \cdot 5$
3. 36 $3^2 \cdot 2^2$
4. 27 3^3
5. 54 $3^3 \cdot 2$
6. 96 $2^5 \cdot 3$
7. 7 7
8. 100 $2^2 \cdot 5^2$
9. 75 $3 \cdot 5^2$

SEE EXAMPLE 2

Find the GCF of each pair of numbers.

10. 12 and 60 12
11. 14 and 49 7
12. 55 and 121 11

SEE EXAMPLE 3

Find the GCF of each pair of monomials.

13. $6x^2$ and $5x^2$ x^2
14. $15y^3$ and $-20y$ $5y$
15. $13q^4$ and $2p^2$ 1

SEE EXAMPLE 4

16. Samantha is making beaded necklaces using 54 glass beads and 18 clay beads. She wants each necklace to have the same number of beads, but each necklace will have only one type of bead. If she puts the greatest possible number of beads on each necklace, how many necklaces can she make? 4

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–24	1
25–27	2
28–30	3
31	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Write the prime factorization of each number.

17. 18 $2 \cdot 3^2$
18. 64 2^6
19. 12 $2^2 \cdot 3$
20. 150 $2 \cdot 3 \cdot 5^2$
21. 17 17
22. 226 $2 \cdot 113$
23. 49 7^2
24. 63 $3^2 \cdot 7$

Find the GCF of each pair of numbers.

25. 36 and 63 9
26. 14 and 15 1
27. 30 and 40 10

Find the GCF of each pair of monomials.

28. $8a^2$ and 11 1
29. $9s$ and $63s^3$ $9s$
30. $-64n^4$ and $24n^2$ $8n^2$

31. José is making fruit-filled tart shells for a party. He has 72 raspberries and 108 blueberries. The tarts will each have the same number of berries. Raspberries and blueberries will not be in the same tart. If he puts the greatest possible number of fruits in each tart, how many tarts can he make? 5

Find the GCF of each pair of products.

32. $3 \cdot 5 \cdot t$ and $2 \cdot 2 \cdot 5 \cdot t \cdot t$ $5t$
33. $-1 \cdot 2 \cdot 2 \cdot x \cdot x$ and $2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot x$ $4x^2$
34. $2 \cdot 2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x$ and $3 \cdot 11$ 11
35. $2 \cdot 5 \cdot n \cdot n \cdot n$ and $-1 \cdot 2 \cdot 3 \cdot n$ $2n$

36. **Write About It** The number 2 is even and is prime. Explain why all other prime numbers are odd numbers.
Possible answer: Even numbers greater than 2 all have 2 as a factor and thus are not prime.

(tr), Sam Dudgeon/HMH; (b), Photographer/collection/JupiterImages

7-1 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 17–27, 40–45, 47–55
Average 17–27, 40–46, 50–55, 68
Advanced 17–27, 40–46, 50–55, 68

If you finished Examples **1–4**
Basic 17–35, 38–59
Average 17–46, 56–59, 60–68 even
Advanced 17–39, 40–46 even, 56–68

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 26, 30, 31, 32, 38

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 16, 31, 38–39, 56, 59

Reason abstractly and quantitatively. Exercises 66–68

Construct viable arguments and critique the reasoning of others. Exercises 36–37

Look for and make use of structure. Exercise 46

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Teaching Tip **Multiple Representations** Suggest to students that they put the information in **Exercise 38** in a table to better organize their work, listing the lengths in increasing order.

ℓ	w	P
1	84	170
2		
3		

Answers

59.

1 ft, 24 ft

$P = 50$ ft;

12 ft, 2 ft

$P = 28$ ft;

8 ft, 3 ft

$P = 22$ ft;

6 ft, 4 ft

$P = 20$ ft;

Patricia should make the pen 4 ft × 6 ft because these dimensions give the shortest perimeter and she will need to buy the least fencing.

38a. 1 × 84, 2 × 42, 3 × 28, 4 × 21, 6 × 14, 7 × 12

37. Critical Thinking The GCF of two numbers is 1. Explain whether this means the two numbers must be prime. No; 2 comp. numbers could share no common factors other than 1.

38. Multi-Step Angelo is making a rectangular floor for a clubhouse with an area of 84 square feet. The length of each side of the floor is a whole number of feet.

a. What are the possible lengths and widths for Angelo's clubhouse floor?

b. What is the minimum perimeter for the clubhouse floor? 38 ft

c. What is the maximum perimeter for the clubhouse floor? 170 ft

39. Music The Cavaliers and the Blue Devils are two of the marching bands that are members of Drum Corps International (DCI). DCI bands are made up of percussionists, brass players, and color guard members who use flags and other props.

In 2004, there were 35 color guard members in the Cavaliers and 40 in the Blue Devils. The two color guards will march in rows with the same number of people in each row without mixing the guards together. If the greatest possible number of people are in each row, how many rows will there be? 15 rows

For each set of numbers, determine which two numbers have a GCF greater than 1, and find that GCF.

40. 11, 12, 14 12 and 14; 2 **41.** 8, 20, 63 8 and 20; 4 **42.** 16, 21, 27 21 and 27; 3

43. 32, 63, 105 63 and 105; 21 **44.** 25, 35, 54 25 and 35; 5 **45.** 35, 54, 72 54 and 72; 18

46. Number Sense The prime factorization of 24 is $2^3 \cdot 3$. Without using a diagram, write the prime factorization of 48. Explain your reasoning. $2^4 \cdot 3$; possible answer: because $48 = 2 \cdot 24$ and $24 = 2^3 \cdot 3$, $48 = 2 \cdot 2^3 \cdot 3 = 2^4 \cdot 3$.

Fill in each diagram. Then write the prime factorization of the number.

47. 72 → 2 • 36; 36 → 2 • 18; 18 → 2 • 9; 9 → 3 • 3 — $2^3 \cdot 3^2$

48. 81 → 3 • 27; 27 → 3 • 9; 9 → 3 • 3 — 3^4

49. 210 → 2 • 105; 105 → 5 • 21; 21 → 3 • 7 — $2 \cdot 3 \cdot 5 \cdot 7$

50. 2 | 56; 2 | 28; 2 | 14; 7 | 7; 1 — $2^3 \cdot 7$

51. 2 | 108; 2 | 54; 3 | 27; 3 | 9; 3 | 3; 1 — $2^2 \cdot 3^3$

52. 2 | 136; 2 | 68; 2 | 34; 17 | 17; 1 — $2^3 \cdot 17$

53. 2 | 48; 2 | 24; 2 | 12; 2 | 6; 3 | 3; 1 — $2^4 \cdot 3$

54. 2 | 140; 2 | 70; 5 | 35; 7 | 7; 1 — $2^2 \cdot 5 \cdot 7$

55. 2 | 40; 2 | 20; 2 | 10; 5 | 5; 1 — $2^3 \cdot 5$

©Thinkstock/Alamy

PRACTICE A
PRACTICE C
PRACTICE B

Write the prime factorization of each number.

1. 18 — $2 \cdot 3^2$
2. 120 — $2^3 \cdot 3 \cdot 5$
3. 56 — $2^3 \cdot 7$
4. 390 — $2 \cdot 3 \cdot 5 \cdot 13$
5. 144 — $2^4 \cdot 3^2$
6. 153 — $3^2 \cdot 17$

Find the GCF of each pair of numbers.

7. 16 and 20 — 4
8. 9 and 36 — 9
9. 15 and 28 — 1
10. 35 and 42 — 7
11. 33 and 66 — 33
12. 100 and 120 — 20
13. 78 and 30 — 6
14. 84 and 42 — 42

Find the GCF of each pair of monomials.

15. $15x^4$ and $35x^2$ — $5x^2$
16. $12p^2$ and $30q^5$ — 6
17. $-6t^2$ and $9t$ — $3t$
18. $27y^2z$ and $45x^2y$ — $9y$
19. $12ab$ and 12 — 12
20. $-8d^3$ and $14d^4$ — $2d^3$
21. $-m^5n^4$ and $3m^5n$ — m^5n
22. $10gh^2$ and $5h$ — $5h$

23. Kirstin is decorating her bedroom wall with photographs. She has 36 photographs of family and 28 photographs of friends. She wants to arrange the photographs in rows so that each row has the same number of photographs, and photographs of family and photographs of friends do not appear in the same row.
   a. How many rows will there be if Kirstin puts the greatest possible number of photographs in each row? 16
   b. How many photographs will be in each row? 4

READING STRATEGIES

There are many vocabulary words related to the concepts of GCF and factoring.

Factor: a number that divides evenly into another number
Ex: All the factors of 12 are 1, 2, 3, 4, 6, and 12.

Prime Number: a number that has exactly two factors, 1 and itself
Ex: The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Factorization: the factors of a number written as a product
Ex: 15 → 1 • 15 and 3 • 5

Prime Factorization: the factors of a number written as a product of prime numbers
Ex: 18 → 2 • 3 • 3 = 2 • 3^2

Greatest Common Factor (GCF): the largest factor of two or more numbers
Ex: The GCF of 12 and 18 is 6. The GCF of $14x^3$ and $8x^2$ is $2x^2$.

Answer the following.

1. List the factors of 28. 1, 2, 4, 7, 14, 28
2. Write the factorization of a^3. a • a • a • a • a
3. List the factors of 16. 1, 2, 4, 8, 16
4. Write the factorization of a^2. a • a
5. What is the GCF of the monomials $28a^5$ and $16a^2$? $4a^2$
6. What is the GCF of the monomials $34m^2$ and $8m$? $2m$
7. Is 33 a prime number? Explain. no; 11 and 3 are factors.

Find the prime factorization of each number.

8. 54 — $2 \cdot 3^2$
9. 150 — $2 \cdot 3 \cdot 5^2$
10. 80 — $2^4 \cdot 5$

RETEACH

A prime number has exactly two factors, itself and 1. The number 1 is not a prime number. To write the **prime factorization** of a number, factor the number into its prime factors only.

Find the prime factorization of 30.

Choose any prime number that is a factor of 30. Then divide.

5 | 30 → 5 | 30, 6 → 5 | 30, 2 • 6 → 5 | 30, 2 • 6, 3 → 5 | 30, 2 • 6, 3 • 3, 1 → 30 = 5 • 2 • 3 • 3

Repeat the process with the quotient.

The prime factorization of 30 is 2 • 3 • 5.

Find the prime factorization of 84.

2 | 84; 2 | 42; 3 | 21; 7 | 7; 1

Check by multiplying: 2 • 2 • 3 • 7 = 84

The prime factorization of 84 is 2 • 2 • 3 • 7 or $2^2 \cdot 3 \cdot 7$.

Fill in the blanks below to find the prime factorization of the given numbers.

1. 2 | 44; 2 | 22; 11 | 11; 1 — $2^2 \cdot 11$
2. 2 | 56; 2 | 28; 2 | 14; 7 | 7; 1 — $2^3 \cdot 7$
3. 3 | 81; 3 | 27; 3 | 9; 3 | 3; 1 — 3^4

Write the prime factorization of each number.

4. 99 — $3^2 \cdot 11$
5. 75 — $3 \cdot 5^2$
6. 84 — $2^2 \cdot 3 \cdot 7$

MULTI-STEP TEST PREP

56. The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in ft/s, a is acceleration in ft/s^2, and t is time in seconds.

a. A toy car begins with a velocity of 2 ft/s and accelerates at 2 ft/s^2. Write an expression for the distance the toy car travels after t seconds. $2t + t^2$

b. What is the GCF of the terms in your expression from part **a**? t

TEST PREP

57. Which set of numbers has a GCF greater than 6?

Ⓐ 18, 24, 36 Ⓑ 30, 35, 40 Ⓒ 11, 29, 37 Ⓓ 16, 24, 48

58. The slope of a line is the GCF of 48 and 12. The y-intercept is the GCF of the slope and 8. Which equation describes the line?

Ⓕ $y = 12x + 4$ Ⓖ $y = 6x + 2$ Ⓗ $y = 4x + 4$ Ⓙ $y = 3x + 1$

59. Extended Response Patricia is making a dog pen in her back yard. The pen will be rectangular and have an area of 24 square feet. Draw and label a diagram that shows all possible whole-number dimensions for the pen. Find the perimeter of each rectangle you drew. Which dimensions should Patricia use in order to spend the least amount of money on fencing materials? Explain your reasoning.

CHALLENGE AND EXTEND

Find the GCF of each set.

60. $4n^3, 16n^2, 8n$ $4n$

61. $27y^3, 18y^2, 81y$ $9y$

62. $100, 25s^5, 50s$ 25

63. $2p^4r, 8p^3r^2, 16p^2r^3$ $2p^2r$

64. $2x^3y, 8x^2y^2, 17xy^3$ xy

65. $8a^4b^3, 4a^3b^3, 12a^2b^3$ $4a^2b^3$

66. Geometry The area of a triangle is 10 in^2. What are the possible whole-number dimensions for the base and height of the triangle? $1 \times 20; 2 \times 10; 4 \times 5; 20 \times 1; 10 \times 2; 5 \times 4$

67. Number Sense The GCF of three different numbers is 7. The sum of the three numbers is 105. What are the three numbers? Possible answer: 21, 35, 49

68. Critical Thinking Find three different *composite* numbers whose GCF is 1. (*Hint:* A composite number has factors other than 1 and itself.) Possible answer: 6, 35, 143

HMH

MULTI-STEP TEST PREP **Exercise 56** involves finding the GCF of terms in an expression for constant acceleration. This exercise prepares students for the Multi-Step Test Prep.

TEST PREP DOCTOR In **Exercise 57,** choice **C** can be eliminated because it lists only prime numbers.

Journal

Have students explain how to use the prime factorization of 36 to find *all* of the factors of 36.

ALTERNATIVE ASSESSMENT

Have students create pairs of numbers or monomials that have the following GCFs: 15, 1, 7, $3x^2$, $2xy$, and $8b$.

Power Presentations with PowerPoint®

7-1 Lesson Quiz

Write the prime factorization of each number.

1. 50 $2 \cdot 5^2$

2. 84 $2^2 \cdot 3 \cdot 7$

Find the GCF of each pair of numbers.

3. 18 and 75 3

4. 20 and 36 4

Find the GCF of each pair of monomials.

5. $12x$ and $28x^3$ $4x$

6. $27x^2$ and $45x^3y^2$ $9x^2$

7. Cindi is planting a rectangular flower bed with 40 orange flowers and 28 yellow flowers. She wants to plant them so that each row will have the same number of plants but of only one color. How many rows will Cindi need if she puts the greatest possible number of plants in each row? 17

Also available online

PROBLEM SOLVING

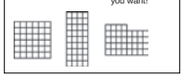
Write the correct answer.

1. Eloise saved all her awards from school. She has 18 athletic awards and 27 academic awards. Eloise wants to display the two types separately but in rows of equal length. Determine the greatest number of awards Eloise can put in each row. Then determine the total number of rows.
   9 awards in each row; total of 5 rows
2. Parker is preparing snacks for the children at a day camp. He has 48 carrot sticks and 36 apple slices. Find the number of identical snacks he can prepare if he puts as much food into each snack as possible without any leftovers. Then describe the snack.
   12 snacks of 4 carrot sticks and 3 apple slices
3. Matías and Hannah are responsible for the centerpieces on the buffet tables at the school dance. They have 6 dozen carnations, 80 lilies, and 64 rosebuds. All the centerpieces must be identical. Determine the greatest number of centerpieces Matías and Hannah can make if they use all the flowers. Then describe the centerpiece.
   8 centerpieces; 9 carnations, 10 lilies, 8 rosebuds
4. Ms. Thompson has 120 same-sized tiles with which to make a rectangular design on her patio floor. 54 of the tiles are green; the rest are blue. To keep the rectangular shape, all rows must have the same number of tiles. If she wants each row to have the same number of blue and green tiles, how many rows will there be if she wants to use all the tiles and wants the rows to be as long as possible?
   6 rows

Part of an ad for interlocking foam squares is shown below. Use it to answer questions 5–7. Select the best answer.

5. A class arranges one package into a rectangle with dimensions other than those shown. Which could have been the dimensions?
   A 2 × 18 C 4 × 8 B 3 × 16 D 5 × 6
6. A teacher has 2 packages of red, 6 packages of blue, and half a package of yellow squares. He wants to build a rectangle so that each row is the same color. What can be the maximum number of squares per row?
   F 2 **H** 18 G 6 J 36
7. In problem 6, how many rows will be blue?
   A 2 C 6 B 3 **D** 12

Each package comes with 36 foam squares that interlock for a safe, colorful floor mat! You can make a ... square, rectangle, or any shape you want!



CHALLENGE

There are many reasons why you might need to factor a number. Small numbers are easily factorable, and you can write their factors with a small amount of work. For large numbers, such as 180, you can use the method shown at right. Notice that the method is based on successive division by small prime numbers. First try 2 as a divisor, then try 3, and so on. From the division shown at right, you can write the prime factorization of 180 as follows.

$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 3^2 5$

2 | 180
2 | 90
3 | 45
3 | 15
5 | 5
1

Using the method shown above, find the prime factorization of each number.

1. 825 $3^1 5^2 11^1$
2. 1575 $3^2 5^2 7^1$
3. 1024 2^{10}
4. 25,200 $2^4 3^2 5^2 7^1$

Once you have written two large numbers by using their prime factorization, you can easily find their common factors.

List the distinct prime-number factors of each pair of numbers. If the numbers have no common factors other than 1, write *relatively prime.*

5. $2^2 3^5 7^2$ and $2^1 3^3 5^3 7^1$ **2, 3, 5, and 7**
6. $2^3 3^1 11^2 13^5$ and $2^1 11^3 19^2$ **2 and 11**
7. $2^{105} 5^6 23^2 53^2$ and $3^1 7^1 17^5 19^4$ **relatively prime**
8. $5^3 3^1 17^2 31^5$ and $2^1 11^3 5^3 3^1 17^2 31^5$ **3, 5, 17, and 31**

Once you have identified which prime numbers are common factors of two numbers, then find the highest powers of those prime factors that are common factors. The product of such powers gives the greatest common factor (GCF) of the two numbers.

Find the greatest common factor of each pair of numbers.

9. the pair in Exercise 5 $2^1 3^2 5^2 7^2$
10. the pair in Exercise 6 $2^1 11^2$
11. the pair in Exercise 7 **1**
12. the pair in Exercise 8 $3^2 5^6 17^2 31^5$
13. 3,293,136 and 3,778,488 $2^3 3^4 7^1$
14. 2,662,000 and 26,159,679 **1**

Organizer

Use with the lesson Factoring by GCF

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use algebra tiles to model and factor polynomials.

Materials: algebra tiles

Online Edition

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

If the algebra tiles (MK) modeling a polynomial cannot be arranged into a rectangle, then the polynomial cannot be factored.

Ask students if they see another way to make a rectangle for $4x + 8$.

Close

Key Concept

You can factor polynomials by arranging tiles into a rectangle and finding expressions for the rectangle's length and width.

Assessment

Journal Have students explain how to model and factor polynomials with algebra tiles.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

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7-2 Algebra LAB

Model Factoring

You can use algebra tiles to write a polynomial as the product of its factors. This process is called *factoring*. Factoring is the reverse of multiplying.

Use with Factoring by GCF

KEY

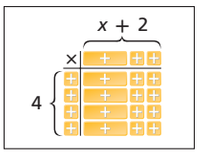
+ = 1
− = −1
+ = x
− = −x
+ = x^2

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

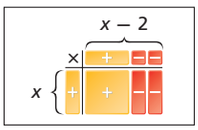
CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Activity

Use algebra tiles to factor $4x + 8$.

MODEL	ALGEBRA
Model 4x + 8.	$4x + 8$
 *Arrange the tiles into a rectangle. The total area represents 4x + 8. The length and width represent the factors. The rectangle has a width of x + 2 and a length of 4.*	$4x + 8 = 4(x + 2)$

Use algebra tiles to factor $x^2 - 2x$.

MODEL	ALGEBRA
Model $x^2 - 2x$.	$x^2 - 2x$
 *Arrange the tiles into a rectangle. The total area represents $x^2 - 2x$. The length and width represent the factors. The rectangle has a width of $x - 2$ and a length of x.*	$x^2 - 2x = x(x - 2)$

Try This

Use algebra tiles to factor each polynomial.

1. $3x + 9$ $3(x + 3)$
2. $2x + 8$ $2(x + 4)$
3. $4x - 12$ $4(x - 3)$
4. $3x - 12$ $3(x - 4)$
5. $2x^2 + 2x$ $2x(x + 1)$
6. $x^2 + 4x$ $x(x + 4)$
7. $x^2 - 3x$ $x(x - 3)$
8. $2x^2 - 4x$ $2x(x - 2)$

COMMON CORE

7-2 Factoring by GCF

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objective
Factor polynomials by using the greatest common factor.

Why learn this?
You can determine the dimensions of a solar panel by factoring an expression representing the panel's area. (See Example 2.)

Recall that the Distributive Property states that $ab + ac = a(b + c)$. The Distributive Property allows you to factor out the GCF of the terms in a polynomial to write a factored form of the polynomial.

A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. The expression $2(3x - 4x)$ is not fully factored because the terms in the parentheses have a common factor of x.

EXAMPLE 1 Factoring by Using the GCF

Factor each polynomial. Check your answer.

A $4x^2 - 3x$

$4x^2 = 2 \cdot 2 \cdot x \cdot x$
$3x = 3 \cdot x$
x — *Find the GCF.*

The GCF of $4x^2$ and $3x$ is x.

$4x(x) - 3(x)$ — *Write terms as products using the GCF as a factor.*

$x(4x - 3)$ — *Use the Distributive Property to factor out the GCF.*

Check $x(4x - 3)$ — *Multiply to check your answer.*
$4x^2 - 3x$ ✓ *The product is the original polynomial.*

B $10y^3 + 20y^2 - 5y$

$10y^3 = 2 \cdot 5 \cdot y \cdot y \cdot y$
$20y^2 = 2 \cdot 2 \cdot 5 \cdot y \cdot y$
$5y = 5 \cdot y$
$5 \cdot y = 5y$ — *Find the GCF.*

The GCF of $10y^3$, $20y^2$, and $5y$ is $5y$.

$2y^2(5y) + 4y(5y) - 1(5y)$ — *Write terms as products using the GCF as a factor.*

$5y(2y^2 + 4y - 1)$ — *Use the Distributive Property to factor out the GCF.*

Check $5y(2y^2 + 4y - 1)$ — *Multiply to check your answer.*
$10y^3 + 20y^2 - 5y$ ✓ *The product is the original polynomial.*

Writing Math
Aligning common factors can help you find the greatest common factor of two or more terms.

Photo Disc/Getty Images

7-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Factor polynomials by using the greatest common factor.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Simplify.

1. $2(w + 1)$ $2w + 2$
2. $3x(x^2 - 4)$ $3x^3 - 12x$

Find the GCF of each pair of monomials.

3. $4h^2$ and $6h$ $2h$
4. $13p$ and $26p^5$ $13p$

Also available online

Math Fact

Around 300 B.C.E., Euclid developed a way of finding the GCF of two numbers by writing the smaller number and the difference of the two numbers as a pair and repeating this process until the two numbers are equal. The GCF of 36 and 60 is found by writing (36, 60); (36, 24); (24, 12); (12, 12). The GCF is 12.

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

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1 Introduce

EXPLORATION

Factoring by GCF

In this Exploration, you will need to use the Distributive Property. Recall that the Distributive Property states that $a(b + c) = ab + ac$.

1. The table shows the factored form of several polynomials. Use the Distributive Property to multiply the factors and write the resulting polynomial in the middle column of the table.

Factored Form	Polynomial	GCF of Terms
$3(x + 2)$	$3x + 6$	3
$x^2(x - 5)$	$x^3 - 5x^2$	x^2
$ab(a + 2b)$	$a^2b + 2ab^2$	ab
$5y^2(y^3 - 8)$	$5y^5 - 40y^2$	$5y^2$
$3a(a^2 + b + 1)$	$3a^3 + 3ab + 3a$	$3a$

2. For each polynomial in the table, find the GCF of the terms. Write the GCF of the terms in the right-hand column of the table.

THINK AND DISCUSS

3. **Describe** what you notice about the GCF of the terms and the factored form of each polynomial.
4. **Explain** whether you think $2(x^2 + x)$ is the factored form of the polynomial $2x^2 + 2x$.

3. The GCF of the terms is factored out in the factored form of the polynomial.

Motivate

Write 6 + 8 on the board. Ask students what the greatest common factor of 6 and 8 is. 2 Then rewrite 6 + 8 with 6 and 8 expressed as products of 2 and one other factor: $2 \cdot 3 + 2 \cdot 4$. Show how to use the Distributive Property to write $2(3 + 4)$.

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

Factor each polynomial. Check your answer.

A. $2x^2 - 4$ $2(x^2 - 2)$

B. $8x^3 - 4x^2 - 16x$ $4x(2x^2 - x - 4)$

C. $-14x - 12x^2$ $-2x(7 + 6x)$

D. $3x^3 + 2x^2 - 10$ cannot be factored

Example 2

The area of a court for the game squash is $(9x^2 + 6x)$ m^2. Factor this polynomial to find possible expressions for the dimensions of the squash court. Possible answer: $3x$ m and $(3x + 2)$ m

Also available online

Factor each polynomial. Check your answer.

C $-12x - 8x^2$

$-1(12x + 8x^2)$ *Both coefficients are negative. Factor out −1.*

$12x = 2 \cdot 2 \cdot 3 \cdot x$
$8x^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x$
$2 \cdot 2 \cdot x = 4x$ *Find the GCF. The GCF of $12x$ and $8x^2$ is $4x$.*

$-1[3(4x) + 2x(4x)]$ *Write each term as a product using the GCF.*

$-1[4x(3 + 2x)]$ *Use the Distributive Property to factor out the GCF.*

$-1(4x)(3 + 2x)$

$-4x(3 + 2x)$

Check

$-4x(3 + 2x) = -12x - 8x^2$ ✓ *Multiply to check your answer.*

D $5x^2 + 7$

$5x^2 = 5 \cdot x \cdot x$
$7 = 7$ *Find the GCF.*

$5x^2 + 7$ *There are no common factors other than 1.*

The polynomial cannot be factored.

Caution!

When you factor out −1 as the first step, be sure to include it in all the other steps as well.

CHECK IT OUT!

Factor each polynomial. Check your answer.

1a. $5b + 9b^3$ $b(5 + 9b^2)$ **1b.** $9d^2 - 8^2$ cannot be factored

1c. $-18y^3 - 7y^2$ $-y^2(18y + 7)$ **1d.** $8x^4 + 4x^3 - 2x^2$ $2x^2(4x^2 + 2x - 1)$

To write expressions for the length and width of a rectangle whose area is expressed as a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

EXAMPLE 2 *Science Application*

Mandy's calculator is powered by solar energy. The area of the solar panel is $(7x^2 + x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

$A = 7x^2 + x$ *The GCF of $7x^2$ and x is x.*

$= 7x(x) + 1(x)$ *Write each term as a product using the GCF as a factor.*

$= x(7x + 1)$ *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the solar panel are x cm and $(7x + 1)$ cm.

CHECK IT OUT!

2. What if...? The area of the solar panel on another calculator is $(2x^2 + 4x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel. $2x$ cm; $(x + 2)$ cm

Victoria Smith/HMH

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you find a common factor?

EXAMPLE 2

- How could factoring a binomial help you with a problem about area?

Teaching Tip **Inclusion** In **Example 1C,** students either factor out −1, or −1 and another positive factor. Remind them that in either case, they need to change the sign of the terms in the parentheses.

2 Teach

Guided Instruction

Show students how the Distributive Property can be used to write $a(b + c)$ as $ab + ac$ and $ab + ac$ as $a(b + c)$. When factoring out common monomials and determining the remaining factors, have students determine the coefficient part first and then the variable part. Encourage students to check their work at each step, mentally "redistributing" the common factor to see if it matches the line in the previous step.

Reaching All Learners

Through Cognitive Strategies

In **Example 5,** students learn to factor with opposites. To help students learn to recognize opposite binomials, work through the first row of the following chart with students. Then have students complete the chart on their own.

	Opposites		
$x - 3$	$-1(x - 3)$ →	$-x + 3$ →	$3 - x$
$8 - a$	$-1(8 - a)$	$-8 + a$	$a - 8$
$m - n$	$-1(m - n)$	$-m + n$	$n - m$
$y^2 - 6$	$-1(y^2 - 6)$	$-y^2 + 6$	$6 - y^2$

Sometimes the GCF of terms is a binomial. This GCF is called a *common binomial factor*. You factor out a common binomial factor the same way you factor out a monomial factor.

EXAMPLE 3 Factoring Out a Common Binomial Factor

Factor each expression.

A $7(x-3)-2x(x-3)$

$7(x-3)-2x(x-3)$ — *$(x-3)$ is a common binomial factor.*

$(x-3)(7-2x)$ — *Factor out $(x-3)$.*

B $-t(t^2+4)+(t^2+4)$

$-t(t^2+4)+(t^2+4)$ — *(t^2+4) is a common binomial factor.*

$-t(t^2+4)+1(t^2+4)$ — *$(t^2+4)=1(t^2+4)$*

$(t^2+4)(-t+1)$ — *Factor out (t^2+4).*

C $9x(x+4)-5(4+x)$

$9x(x+4)-5(4+x)$ — *$(x+4)=(4+x)$, so $(x+4)$ is a common binomial factor.*

$9x(x+4)-5(x+4)$

$(x+4)(9x-5)$ — *Factor out $(x+4)$.*

D $-3x^2(x+2)+4(x-7)$ — *There are no common factors.*

$-3x^2(x+2)+4(x-7)$

The expression cannot be factored.

CHECK IT OUT! **Factor each expression.**

3a. $4s(s+6)-5(s+6)$ — $(4s-5)(s+6)$

3b. $7x(2x+3)+(2x+3)$ — $(7x+1)(2x+3)$

3c. $3x(y+4)-2y(x+4)$ — cannot be factored

3d. $5x(5x-2)-2(5x-2)$ — $(5x-2)^2$

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can make two groups and factor out the GCF from each group.

EXAMPLE 4 Factoring by Grouping

Factor each polynomial by grouping. Check your answer.

A $12a^3-9a^2+20a-15$

$(12a^3-9a^2)+(20a-15)$ — *Group terms that have a common number or variable as a factor.*

$3a^2(4a-3)+5(4a-3)$ — *Factor out the GCF of each group.*

$3a^2(4a-3)+5(4a-3)$ — *$(4a-3)$ is a common factor.*

$(4a-3)(3a^2+5)$ — *Factor out $(4a-3)$.*

Check $(4a-3)(3a^2+5)$ — *Multiply to check your solution.*

$4a(3a^2)+4a(5)-3(3a^2)-3(5)$

$12a^3+20a-9a^2-15$

$12a^3-9a^2+20a-15$ ✓ — *The product is the original polynomial.*

COMMON ERROR ALERT

Students may forget that a binomial written without a coefficient actually has a coefficient of 1. Suggest that students write in the coefficient of 1 so that they don't forget it is there.

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Additional Examples

Example 3

Factor each expression.

A. $5(x+2)+3x(x+2)$
$(x+2)(5+3x)$

B. $-2b(b^2+1)+(b^2+1)$
$(b^2+1)(-2b+1)$

C. $4z(z^2-7)+9(2z^3+1)$
cannot be factored

Example 4

Factor each polynomial by grouping. Check your answer.

A. $6h^4-4h^3+12h-8$
$(3h-2)(2h^3+4)$

B. $5y^4-15y^3+y^2-3y$
$(y-3)(5y^3+y)$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How is the Distributive Property used in these examples?

EXAMPLE 4

- How do you determine which terms to group?
- How can you check your answer?

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Additional Examples

Example 5

Factor $2x^3 - 12x^2 + 18 - 3x$ by grouping. $(x - 6)(2x^2 - 3)$

Also available online

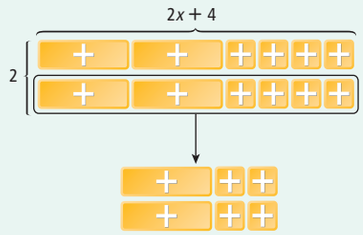
Questioning Strategies

EXAMPLE 5

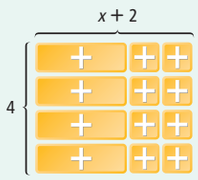
- How can you recognize opposite binomials?
- Does it matter which pair of terms has –1 as a factor? Explain.

Teaching Tip **Visual Cues** Students can check their answers using algebra tiles (MK). If an expression is not factored fully, each row of the model can be rearranged into a smaller rectangle.

If $4x + 8$ is factored as $2(2x + 4)$, then each row can still form another rectangle:



When $4x + 8$ is completely factored as $4(x + 2)$, the rows cannot form another rectangle:



Factor each polynomial by grouping. Check your answer.

B $9x^3 + 18x^2 + x + 2$

$(9x^3 + 18x^2) + (x + 2)$ *Group terms.*

$9x^2(x + 2) + 1(x + 2)$ *Factor out the GCF of each group.*

$9x^2(x + 2) + 1(x + 2)$ *$(x + 2)$ is a common factor.*

$(x + 2)(9x^2 + 1)$ *Factor out $(x + 2)$.*

Check $(x + 2)(9x^2 + 1)$ *Multiply to check your solution.*

$x(9x^2) + x(1) + 2(9x^2) + 2(1)$

$9x^3 + x + 18x^2 + 2$

$9x^3 + 18x^2 + x + 2$ ✓ *The product is the original polynomial.*

Check It Out! Factor each polynomial by grouping. Check your answer.

4a. $6b^3 + 8b^2 + 9b + 12$ $(2b^2 + 3)(3b + 4)$

4b. $4r^3 + 24r + r^2 + 6$ $(4r + 1)(r^2 + 6)$

Helpful Hint

If two quantities are opposites, their sum is 0.

$(5 - x) + (x - 5)$
$5 - x + x - 5$
$(-x + x) + (5 - 5)$
$0 + 0$
0

Recognizing opposite binomials can help you factor polynomials. The binomials $(5 - x)$ and $(x - 5)$ are opposites. Notice $(5 - x)$ can be written as $-1(x - 5)$.

$-1(x - 5) = (-1)(x) + (-1)(-5)$ *Distributive Property*

$= -x + 5$ *Simplify.*

$= 5 - x$ *Commutative Property of Addition*

So, $(5 - x) = -1(x - 5)$.

EXAMPLE 5 Factoring with Opposites

Factor $3x^3 - 15x^2 + 10 - 2x$ by grouping.

$3x^3 - 15x^2 + 10 - 2x$

$(3x^3 - 15x^2) + (10 - 2x)$ *Group terms.*

$3x^2(x - 5) + 2(5 - x)$ *Factor out the GCF of each group.*

$3x^2(x - 5) + 2(-1)(x - 5)$ *Write $(5 - x)$ as $-1(x - 5)$.*

$3x^2(x - 5) - 2(x - 5)$ *Simplify. $(x - 5)$ is a common factor.*

$(x - 5)(3x^2 - 2)$ *Factor out $(x - 5)$.*

Check It Out! Factor each polynomial by grouping. Check your answer.

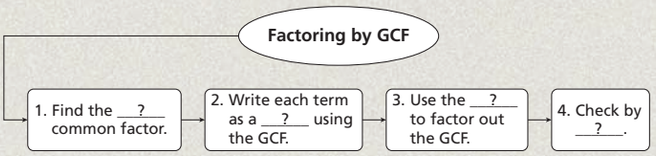
5a. $15x^2 - 10x^3 + 8x - 12$ $(5x^2 - 4)(3 - 2x)$

5b. $8y - 8 - x + xy$ $(8 + x)(y - 1)$

THINK AND DISCUSS

Know it! Note

1. Explain how finding the GCF of monomials helps you factor a polynomial.
2. **GET ORGANIZED** Copy and complete the graphic organizer.



3 Close

Summarize

Ask students whether each of the following can be factored further. If yes, have them factor the polynomial.

$2x^5 - 7x^3$ yes; $x^3(2x^2 - 7)$

$3x^4 + 13y^4 + 3z^4$ no

$p(2p - 1) - 2(p - 1)$ no

$g^2(g - 8) + 6(8 - g)$

yes; $(g - 8)(g^2 - 6)$

$14y^5 + 21$ yes; $7(2y^5 + 3)$

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. When you know the GCF of the monomials in a polynomial, you can factor out the GCF from each monomial to factor the polynomial.
2. See Additional Answers.

Teaching Tip **Multiple Representations** Some factorizations may be written differently but still be correct. For instance, **Check It Out 5a** could also be written as $(4 - 5x^2)(2x - 3)$.

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Factor each polynomial. Check your answer.

1. $15a - 5a^2$ $5a(3 - a)$
2. $10g^3 - 3g$ $g(10g^2 - 3)$
3. $-35x + 42$ $7(-5x + 6)$
4. $-4x^2 - 6x$ $-2x(2x + 3)$
5. $12h^4 + 8h^2 - 6h$
6. $3x^2 - 9x + 3$
7. $9m^2 + m$
8. $14n^3 + 7n + 7n^2$
9. $36f + 18f^2 + 3$
10. $-15b^2 + 7b$

SEE EXAMPLE 2

11. **Physics** A model rocket is fired vertically into the air at 320 ft/s. The expression $-16t^2 + 320t$ gives the rocket's height after t seconds. Factor this expression. $16t(-t + 20)$

SEE EXAMPLE 3

Factor each expression.

12. $5(m - 2) - m(m - 2)$ $(5 - m)(m - 2)$
13. $2b(b + 3) + 5(b + 3)$ $(2b + 5)(b + 3)$
14. $4(x - 3) - x(y + 2)$ cannot be factored

Factor each polynomial by grouping. Check your answer.

SEE EXAMPLE 4

15. $x^3 + 4x^2 + 2x + 8$
16. $6x^3 + 4x^2 + 3x + 2$
17. $4b^3 - 6b^2 + 10b - 15$
18. $2m^3 + 4m^2 + 6m + 12$
19. $7r^3 - 35r^2 + 6r - 30$
20. $10a^3 + 4a^2 + 5a + 2$

SEE EXAMPLE 5

21. $2r^2 - 6r + 12 - 4r$
22. $6b^2 - 3b + 4 - 8b$
23. $14q^2 - 21q + 6 - 4q$
24. $3r - r^2 + 2r - 6$
25. $2m^3 - 6m^2 + 9 - 3m$
26. $6a^3 - 9a^2 - 12 + 8a$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–35	1
36	2
37–42	3
43–48	4
49–54	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

37. cannot be factored

Factor each polynomial. Check your answer.

27. $9y^2 + 45y$ $9y(y + 5)$
28. $36d^3 + 24$ $12(3d^3 + 2)$
29. $-14x^4 + 5x^2$ $x^2(-14x^2 + 5)$
30. $-15f - 10f^2$
31. $-4d^4 + d^3 - 3d^2$
32. $14x^3 + 63x^2 - 7x$
33. $21c^2 + 14c$
34. $33d^3 + 22d + 11$
35. $-5g^3 - 15g^2$
36. **Finance** After t years, the amount of money in a savings account that earns simple interest is $P + Prt$, where P is the starting amount and r is the yearly interest rate. Factor this expression. $P(1 + rt)$

Factor each expression.

37. $6a(a - 2) - 5b(b + 4)$
38. $-4x(x + 2) + 9(x + 2)$ $(-4x + 9)(x + 2)$
39. $6y(y - 7) + (y - 7)$ $(6y + 1)(y - 7)$
40. $a(x - 3) + 2b(x - 3)$ $(a + 2b)(x - 3)$
41. $-3(2 + b) + 4b(b + 2)$ $(-3 + 4b)(b + 2)$
42. $5(3x - 2) + x(3x - 2)$ $(5 + x)(3x - 2)$

Factor each polynomial by grouping. Check your answer.

43. $2a^3 - 8a^2 + 3a - 12$
44. $x^3 + 3x^2 + 5x + 15$
45. $6x^3 + 18x^2 + x + 3$
46. $7x^3 + 2x^2 + 28x + 8$
47. $n^3 - 2n^2 + 5n - 10$
48. $10b^3 - 16b^2 + 25b - 40$
49. $2m^3 - 2m^2 + 3 - 3m$
50. $2d^3 - d^2 - 3 + 6d$
51. $6f^3 - 8f^2 + 20 - 15f$
52. $5k^2 - k^3 + 3k - 15$
53. $b^3 - 2b - 8 + 4b^2$
54. $20 - 15x - 6x^2 + 8x$

Tony Freeman/Photoedit

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3
- **Basic** 27–42, 55–58
- **Average** 27–42, 55, 57, 74, 76
- **Advanced** 27–42, 55, 57, 74, 76

If you finished Examples 1–5
- **Basic** 27–54, 55–62, 66, 68, 70–73
- **Average** 27–39, 40–54 even, 55–76
- **Advanced** 28–32 even, 39, 40–54 even, 55–69, 71–80

Homework Quick Check

Quickly check key concepts.
Exercises: 28, 39, 40, 44, 54, 66

Answers

5. $2h(6h^3 + 4h - 3)$
6. $3(x^2 - 3x + 1)$
7. $m(9m + 1)$
8. $7n(2n^2 + 1 + n)$
9. $3(12f + 6f^2 + 1)$
10. $b(-15b + 7)$
15. $(x^2 + 2)(x + 4)$
16. $(2x^2 + 1)(3x + 2)$
17. $(2b^2 + 5)(2b - 3)$
18. $2(m + 2)(m^2 + 3)$
19. $(7r^2 + 6)(r - 5)$
20. $(2a^2 + 1)(5a + 2)$
21. $2(r - 2)(r - 3)$
22. $(3b - 4)(2b - 1)$
23. $(7q - 2)(2q - 3)$
24. $(2 - r)(r - 3)$
25. $(2m^2 - 3)(m - 3)$
26. $(3a^2 + 4)(2a - 3)$
30. $-5f(3 + 2f)$
31. $-d^2(4d^2 - d + 3)$
32. $7x(2x^2 + 9x - 1)$
33. $7c(3c + 2)$
34. $11(3d^3 + 2d + 1)$
35. $-5g^2(g + 3)$
43. $(2a^2 + 3)(a - 4)$
44. $(x^2 + 5)(x + 3)$
45. $(6x^2 + 1)(x + 3)$
46. $(x^2 + 4)(7x + 2)$
47. $(n^2 + 5)(n - 2)$
48. $(2b^2 + 5)(5b - 8)$
49. $(2m^2 - 3)(m - 1)$
50. $(d^2 + 3)(2d - 1)$
51. $(2f^2 - 5)(3f - 4)$
52. $(k^2 - 3)(5 - k)$
53. $(b^2 - 2)(b + 4)$
54. $(5 + 2x)(4 - 3x)$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 11, 36, 63, 66, 68, 80

Construct viable arguments and critique the reasoning of others. Exercises 64–65, 67, 69–70

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 68** involves discovering the Zero Product Property. This exercise prepares students for the Multi-Step Test Prep

Answers

63b. $100x^3 + 200x^2 + 400x + 800$

65. $(3a - 3b) - (4a - 4b)$
$3(a - b) - 4(a - b)$
$(3 - 4)(a - b)$
$-1(a - b)$
$(b - a)$;
$(3a - 4a) - (3b - 4b)$
$a(3 - 4) - b(3 - 4)$
$(a - b)(3 - 4)$
$(a - b)(-1)$
$(b - a)$

68a. Either a, b, or both must equal 0.

b. The product of t and $3 - t$ is 0, so at least one of the factors must be 0.

Fill in the missing part of each factorization.

55. $16v + 12v^2 = 4v(4 + \mathbf{3v})$

56. $15x - 25x^2 = 5x(3 - \mathbf{5x})$

57. $-16k^3 - 24k^2 = -8k^2(\mathbf{2k} + 3)$

58. $-x - 10 = -1(\mathbf{x} + 10)$

Copy and complete the table.

	Polynomial	Number of Terms	Name	Completely Factored Form
	$3y + 3x + 9$	3	trinomial	$3(y + x + 3)$
59.	$x^2 + 5x$	2	binomial	$x(x + 5)$
60.	$28c^2 - 49c$	2	binomial	$7c(4c - 7)$
61.	$a^4 + a^3 + a^2$	3	trinomial	$a^2(a^2 + a + 1)$
62.	$36 + 99r - 40r^2 - 110r^3$	4	polynomial	$(11r + 4)(-10r^2 + 9)$

63. Personal Finance The final amount of money earned by a certificate of deposit (CD) after n years is $P(1 + r)^n$, where P is the original amount contributed and r is the interest rate as a decimal.

Year	Amount of CD
2004	$100.00
2005	$200.00
2006	$400.00

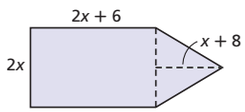
Justin's aunt purchased three CDs with the same interest rate to help him pay for college. The table shows the amount of the CD she purchased each year. In 2007, she paid $800.00 directly to the college.

a. Let $x = 1 + r$. Write expressions in terms of x for the value of the CDs purchased in 2004, 2005, and 2006 when Justin started college in 2007. $100x^3$; $200x^2$; $400x$

b. Write a polynomial in terms of x to represent the total value of the CDs purchased in 2004, 2005, and 2006 plus the amount paid to the college in 2007.

c. Factor the polynomial in part **b** by grouping. Evaluate the factored form of the polynomial when the interest rate is 9%. (*Hint*: Remember that $x = 1 +$ the interest rate expressed as a decimal.) $100(x^2 + 4)(x + 2)$; $1603.12

64. Write About It Describe how to find the area of the figure shown. Show each step and write your answer in factored form. $5x(x + 4)$



65. Critical Thinking Show two methods of factoring the expression $3a - 3b - 4a + 4b$.

66. Geometry The area of a triangle is $\frac{1}{2}(x^3 - 2x + 2x^2 - 4)$. The height h is $x + 2$. Write an expression for the base b of the triangle. (*Hint:* Area of a triangle $= \frac{1}{2}bh$) $x^2 - 2$

67. Write About It Explain how you know when two binomials are opposites.
The sum of opposite binomials is 0.

MULTI-STEP TEST PREP

68. a. The Multiplication Property of Zero states that the product of any number and 0 is 0. What must be true about either a or b to make $ab = 0$?

b. A toy car's distance in feet from the starting point is given by the equation $d = t(3 - t)$. Explain why $t(3 - t) = 0$ means that either $t = 0$ or $3 - t = 0$.

c. When $d = 0$, the car is at the starting point. Use the fact that $t = 0$ or $3 - t = 0$ when $d = 0$ to find the two times when the car is at the starting point.
$t = 0$ and $t = 3$

HMH

PRACTICE A
PRACTICE C
PRACTICE B

Factor each polynomial. Check your answer.

1. $8c^2 + 7c$ — $c(8c + 7)$
2. $3n^3 + 12n^2$ — $3n^2(n + 4)$
3. $15x^5 - 18x$ — $3x(5x^4 - 6)$
4. $-8s^4 + 20s^3 - 28$ — $4(-2s^4 + 5t^3 - 7)$
5. $6n^5 + 18n^4 - 24n$ — $6n(n^5 + 3n^3 - 4)$
6. $-5m^4 - 5m^3 + 5m^2$ — $5m^2(-m^2 - m + 1)$
7. A ball is hit vertically into the air using a paddle at a speed of 32 ft/sec. The expression $-16t^2 + 32t$ gives the ball's height after t seconds. Factor this expression. — $16t(-t + 2)$
8. The area of Margo's laptop computer screen is $12x^2 + 3x$ in^2. Factor this polynomial to find expressions for the dimensions of her computer screen. — $3x$ and $4x + 1$

Factor each expression.

9. $3m(m + 5) + 4(m + 5)$ — $(m + 5)(3m + 4)$
10. $16b(b - 3) + (b - 3)$ — $(b - 3)(16b + 1)$

Factor each polynomial by grouping.

11. $2x^3 + 8x^2 + 3x + 12$ — $(x + 4)(2x^2 + 3)$
12. $4n^3 + 3n^2 + 4n + 3$ — $(4n + 3)(n^2 + 1)$
13. $10d^2 - 6d + 35d - 21$ — $(5d - 3)(2d + 7)$
14. $12n^3 - 15n^2 - 8n + 10$ — $(4n - 5)(3n^2 - 2)$
15. $5b^4 - 15b^3 + 3 - b$ — $(b - 3)(5b^3 - 1)$
16. $t^3 - 5t^2 + 10 - 2t$ — $(t^2 - 2)(t - 5)$

READING STRATEGIES

To factor a polynomial by using the GCF, think of using the Distributive Property in reverse. More specifically, follow the procedure shown here.

$15b^3 + 18b^2 - 6b^2$

Step 1: Find the GCF of the coefficients.
15: 1, 3, 5, 15
18: 1, 2, 3, 6, 9, 18
6: 1, 2, 3, 6

Step 2: Identify the lowest degree of the variable.
b^3
b^2
b^1

Step 3: Write the results of Steps 1 and 2 as a product.
$3b^2$

Step 4: Use the Distributive Property to factor out this GCF.
$3b^2(5b^3 + 6b - 2)$

Step 5: Check using multiplication.
$3b^2(5b^3 + 6b - 2) = 15b^5 + 18b^3 - 6b^2$ ✓

Factor each polynomial using the procedure shown above.

1. $4x^3 + 12x^2$ — $4x^2(x + 3)$
2. $30t^4 - 18t$ — $6t(5t^3 - 3)$
3. $9p^2 + p$ — $p(9p + 1)$
4. $28r^4 - 20r^2 - 8$ — $4(7r^4 - 5r^2 - 2)$
5. $30p^8 + 45p^5$ — $15p^5(2p^3 + 3)$
6. $6m^8 - 16m^3 - 6m^2$ — $2m^2(3m^5 - 8m - 3)$

RETEACH

The Distributive Property states: $a(b + c) = ab + ac$
Factoring by GCF reverses the Distributive Property:
$ab + ac = a(b + c)$

Factor $12x^3 + 21x^2 + 15x$. Check your answer.

Step 1: Find the GCF of all the terms in the polynomial.
The factors of $12x^3$ are: 1, 2, **3**, 4, 6, 12, ***x***, *x*, *x*
The factors of $21x^2$ are: 1, **3**, 7, 21, ***x***, *x*
The factors of $15x$ are: 1, **3**, 5, 15, ***x***
} The GCF is **3*x***.

Step 2: Write terms as products using the GCF.
$12x^3 + 21x^2 + 15x$
$(3x)4x^2 + (3x)7x + (3x)5$

Step 3: Use the Distributive Property to factor out the GCF.
$3x(4x^2 + 7x + 5)$

Check:
$3x(4x^2 + 7x + 5) = 12x^3 + 21x^2 + 15x$ ✓

Factor $5(x - 3) + 4x(x - 3)$.

Step 1: Find the GCF of all the terms in the polynomial.
The factors of $5(x - 3)$ are: 5, **(*x* – 3)**
The factors of $4x(x - 3)$ are: 4, *x*, **(*x* – 3)**
} The GCF is **(*x* – 3)**.
The terms are already written as products with the GCF.

Step 2: Use the Distributive Property to factor out the GCF.
$(x - 3)(5 + 4x)$

Factor each polynomial.

1. $20x^2 - 15x$ — $5x(4x - 3)$
2. $44a^2 + 11a$ — $11a(4a + 1)$
3. $24y - 36x$ — $12(2y - 3x)$

Factor each expression.

4. $5x(x + 7) + 2(x + 7)$ — $(x + 7)(5x + 2)$
5. $3a(a + 4) - 2(a + 4)$ — $(a + 4)(3a - 2)$
6. $4y(4y + 1) + (4y + 1)$ — $(4y + 1)^2$

Fill in each blank with a property or definition that justifies the step.

69. $7x^3 + 2x + 21x^2 + 6 = 7x^3 + 21x^2 + 2x + 6$ **a.** ?

$= (7x^3 + 21x^2) + (2x + 6)$ **b.** ?

$= 7x^2(x + 3) + 2(x + 3)$ **c.** ?

$= (x + 3)(7x^2 + 2)$ **d.** ?

69a. Comm. Prop. of Add.
b. Assoc. Prop. of Add.
c. Dist. Prop.
d. Dist. Prop.

70. ///ERROR ANALYSIS/// Which factorization of $3n^3 - n^2$ is incorrect? Explain.

A

$3n^3 - n^2$
$n^2(3n) - n^2(0)$
$n^2(3n - 0)$

B

$3n^3 - n^2$
$n^2(3n) - n^2(1)$
$n^2(3n - 1)$

A is incorrect because $n^2 \neq n^2(0)$.

TEST PREP

71. Which is the complete factorization of $24x^3 - 12x^2$?

Ⓐ $6(4x^3 - 2x^2)$ Ⓑ $12(2x^3 - x^2)$ Ⓒ $12x(2x^2 - x)$ Ⓓ $12x^2(2x - 1)$

72. Which is NOT a factor of $18x^2 + 36x$?

Ⓕ 1 Ⓖ $4x$ Ⓗ $x + 2$ Ⓙ $18x$

73. The area of a rectangle is represented by the polynomial $x^2 + 3x - 6x - 18$. Which of the following could represent the length and width of the rectangle?

Ⓐ Length: $x + 3$; width: $x + 6$
Ⓑ Length: $x - 3$; width: $x - 6$
Ⓒ Length: $x + 3$; width: $x - 6$
Ⓓ Length: $x - 3$; width: $x + 6$

CHALLENGE AND EXTEND

Factor each polynomial.

74. $6a(b^2 - 4a)$ **74.** $6ab^2 - 24a^2$

75. $-72a^2b^2 - 45ab$ $-9ab(8ab + 5)$

76. $-18a^2b^2 + 21ab$ $-3ab(6ab - 7)$

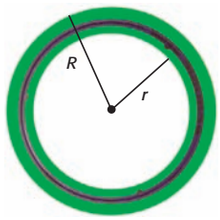
77. $(a + c)(b + d)$ **77.** $ab + bc + ad + cd$

78. $4y^2 + 8ay - y - 2a$

79. $x^3 - 4x^2 + 3x - 12$

78. $(4y - 1)(y + 2a)$
79. $(x^2 + 3)(x - 4)$

80. Geometry The area between two concentric circles is called an *annulus*. The formula for area of an annulus is $A = \pi R^2 - \pi r^2$, where R is the radius of the larger circle and r is the radius of the smaller circle. $A = \pi(R^2 - r^2)$



a. Factor the GCF from the formula for area of an annulus.

b. Use your answer from part **a** to find the area of an annulus with $R = 12$ cm and $r = 5$ cm $A \approx 374$ cm^2

PROBLEM SOLVING

Write the correct answer.

1. The area of a rug, which is shaped like a rectangle, is $4x^2 + 4x$ square feet. Factor this polynomial to find expressions for the 17imensions of the rug.

4x ft; (x + 1) ft

2. The number of customers visiting a local museum since the year 2000 can be modeled by the expression $-3x^2 - 27x + 825$, where x is the number of years since 2000. Factor this polynomial.

$-3(x^2 + 9x - 275)$

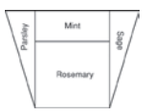
3. The perimeter of a rhombus is $12x + 28$ feet. Factor this expression. Then find the length of one side if $x = 8$. (*Hint:* A rhombus is a parallelogram with four congruent sides.)

4(3x + 7); 31 feet

4. The foundation for a new high school building is rectangular in shape, and the area is $5x^3 + 4x^2 - 10x - 8$ square meters. Factor by grouping to find expressions for the dimensions of the building.

(5x + 4) m; $(x^2 - 2)$ m

The diagram shows four sections of an herb garden. Use the figure to answer questions 5–8. Select the best answer.



5. The section where rosemary grows is square and has an area of $4x^2$ square feet. What is the length of one side?

A x feet C $2x$ feet
B x^2 feet D $4x$ feet

6. Rosemary and mint cover $6x^2 - 2x$ square feet. Assuming the length is adjacent to rosemary, what is the width of the mint section?

F $2x$ feet H $2x - 2$ feet
G $x - 1$ feet J $3x - 1$ feet

7. The parsley and sage sections each have an area of $\frac{1}{2}(3x^2 - 6x - x + 2)$ square feet. Factor $3x^2 - 6x - x + 2$. What are the base and height of each triangular section?

A $2x - 3$ feet; $x + 1$ feet
B $2x - 3$ feet; $x^2 + 1$ feet
C $3x - 1$ feet; $x - 2$ feet
D $3x - 1$ feet; $x^2 - 2$ feet

8. Assuming the side adjacent to mint and rosemary is the base, what is the height of each triangle on which parsley and sage grow?

F $x - 2$ feet
G $x + 1$ feet
H $x^2 + 1$ feet
J $2x$ feet

CHALLENGE

Factoring by grouping requires 4 or more terms. When a polynomial has 3 terms, it is possible to divide the middle term into two terms and factor by grouping. When deciding on which two terms to use, think of terms that will have a common factor with the first and last terms. If that pair doesn't work, change the order of the pairs, or try another pair.

$x^2 + 10x + 21$
$x^2 + 3x + 7x + 21$
$(x^2 + 3x) + (7x + 21)$
$x(x + 3) + 7(x + 3)$
$(x + 3)(x + 7)$

Divide the *b* term into two separate terms. Then factor by grouping.

1. $x^2 + 8x + 12$ 2. $x^2 - 13x + 42$ 3. $2x^2 + 13x + 21$

(x + 6)(x + 2) (x − 6)(x − 7) (2x + 7)(x + 3)

4. $10x^2 + 19x + 6$ 5. $5x^2 + 19x + 12$ 6. $4x^2 - 16x + 15$

(5x + 2)(2x + 3) (5x + 4)(x + 3) (2x − 5)(2x − 3)

When a polynomial has more than 4 terms, it is sometimes possible to factor by grouping.

$(x^3 + 4x^2) + (x^2 + 4x) + (3x + 12)$
$x^2(x + 4) + x(x + 4) + 3(x + 4)$
$(x + 4)(x^2 + x + 3)$

Factor by grouping.

7. $(x^3 + x^2) + (x^2 + x) + (x + 1)$ 8. $(3x^3 - 6x^2) + (4x^2 - 8x) + (5x - 10)$

$(x + 1)(x^2 + x + 1)$ $(x - 2)(3x^2 + 4x + 5)$

Factor by grouping in groups of three. You may need to rearrange the terms.

9. $x^6 + 5x^5 + 7x^4 + 3x^2 + 15x + 21$

$(x^4 + 3)(x^2 + 5x + 7)$

10. $6x^{10} + 24x^9 + 48x^8 + 7x^6 + 28x^5 + 56x^4 + 3x^2 + 12x + 24$

$(6x^8 + 7x^4 + 3)(x^2 + 4x + 8)$

TEST PREP DOCTOR If students have difficulty with **Exercise 71,** remind them that a complete factorization means that the terms *inside* the parentheses have no common factors. That eliminates all options except **D.**

If students chose **H** for **Exercise 72,** they may have forgotten that a binomial can be a factor of a binomial. Remind them that $ab + ac = a(b + c)$. Then point out that 4 is not a factor of 18, which means that **G** is the correct answer.

Journal

Have students use examples to describe how to determine whether a polynomial with four terms can be factored into two binomial expressions.

ALTERNATIVE ASSESSMENT

Have students write three polynomial expressions such that one expression cannot be factored, another has a monomial GCF, and the third has a binomial GCF.

Power Presentations with PowerPoint®

7-2 Lesson Quiz

Factor each polynomial. Check your answer.

1. $16x + 20x^3$ $4x(4 + 5x^2)$

2. $4m^4 - 12m^2 + 8m$ $4m(m^3 - 3m + 2)$

Factor each expression.

3. $7k(k - 3) + 4(k - 3)$ $(k - 3)(7k + 4)$

4. $3y(2y + 3) - 5(2y + 3)$ $(2y + 3)(3y - 5)$

Factor each polynomial by grouping. Check your answer.

5. $2x^3 + x^2 - 6x - 3$ $(2x + 1)(x^2 - 3)$

6. $7p^4 - 2p^3 + 63p - 18$ $(7p - 2)(p^3 + 9)$

7. A rocket is fired vertically into the air at 40 m/s. The expression $-5t^2 + 40t + 20$ gives the rocket's height after t seconds. Factor this expression. $-5(t^2 - 8t - 4)$

Also available online

Organizer

Use with the lesson Factoring $x^2 + bx + c$

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Model and factor trinomials.

Materials: algebra tiles

Online Edition
Algebra Tiles

Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

Remind students that a square is a rectangle, so their algebra tile (MK) arrangements may form a square.

In **Activity 2,** additional *x*-tiles were used because there were originally not enough to fill the empty spaces.

Alternative Approach

Use the transparency mat and transparency algebra tiles (MK).

MATHEMATICAL PRACTICES
Use appropriate tools strategically.

COMMON CORE
CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Learn It Online
State Resources Online

7-3 Algebra LAB

Use with Factoring $x^2 + bx + c$

Model Factorization of Trinomials

You can use algebra tiles to write a trinomial as a product of two binomials. This is called *factoring a trinomial.*

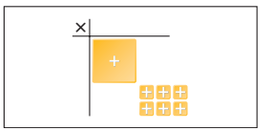
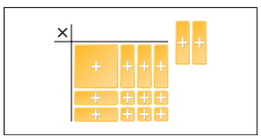
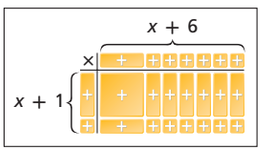
KEY

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Activity 1

Use algebra tiles to factor $x^2 + 7x + 6$.

MODEL	ALGEBRA
Model $x^2 + 7x + 6$.	$x^2 + 7x + 6$
 *Try to arrange all of the tiles in a rectangle. Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.*	$x^2 + 7x + 6 \neq (x + 2)(x + 3)$
 *Arrange the x-tiles so that all the tiles together make one large rectangle.* *This arrangement does not work because two x-tiles are left over.*	
Rearrange the unit tiles to form another rectangle.	
 *Fill in the empty spaces with x-tiles. All 7 x-tiles fit. This is the correct arrangement.* *The total area represents the trinomial. The length and width represent the factors.*	$x^2 + 7x + 6 = (x + 1)(x + 6)$

The rectangle has width $x + 1$ and length $x + 6$. So $x^2 + 7x + 6 = (x + 1)(x + 6)$.

Answers to *Try This*

1. $(x + 1)(x + 1)$
2. $(x + 1)(x + 2)$
3. $(x + 1)(x + 5)$
4. $(x + 3)(x + 3)$
5. $(x + 1)(x + 4)$
6. $(x + 2)(x + 4)$
7. $(x + 2)(x + 3)$
8. $(x + 2)(x + 6)$

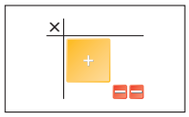
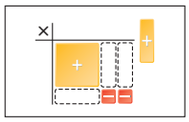
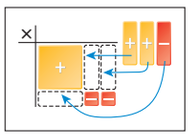
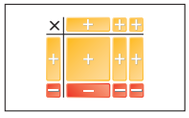
Try This

Use algebra tiles to factor each trinomial.

1. $x^2 + 2x + 1$
2. $x^2 + 3x + 2$
3. $x^2 + 6x + 5$
4. $x^2 + 6x + 9$
5. $x^2 + 5x + 4$
6. $x^2 + 6x + 8$
7. $x^2 + 5x + 6$
8. $x^2 + 8x + 12$

Activity 2

Use algebra tiles to factor $x^2 + x - 2$.

MODEL		ALGEBRA
	Model $x^2 + x - 2$.	$x^2 + x - 2$
	*Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.*	
	*To make a rectangle, you need to fill in the empty spaces, but there aren't enough x-tiles to fill in the empty spaces.*	
	*Add a zero pair. Arrange the x-tiles to complete the rectangle.* *Remember that the product of two positive values is positive and the product of a positive and a negative value is negative.*	
	*The total area represents the trinomial. The length and width represent the factors.*	$x^2 + x - 2 = (x - 1)(x + 2)$

The rectangle has width $x - 1$ and length $x + 2$. So, $x^2 + x - 2 = (x - 1)(x + 2)$.

Try This

9. Why can you add one red $-x$-tile and one yellow x-tile?

Use algebra tiles to factor each trinomial.

10. $x^2 - x - 2$
11. $x^2 - 2x - 3$
12. $x^2 - 5x + 4$
13. $x^2 - 7x + 10$
14. $x^2 - 2x + 1$
15. $x^2 - 6x + 5$
16. $x^2 + 5x - 6$
17. $x^2 + 3x - 4$
18. $x^2 - x - 6$
19. $x^2 + 3x - 10$
20. $x^2 - 2x - 8$
21. $x^2 + x - 12$

Close

Key Concept

You can factor trinomials by arranging the tiles into a rectangle such that the x^x-tile is in the upper left, the unit tiles are in the lower right, and the x-tiles fill in the remaining spaces. The dimensions of the rectangle are the factors.

Assessment

Journal Have students explain how to model and factor trinomial expressions. Explanations should include what to do when there are not enough x-tiles to fill the empty spaces.

Answers to *Try This*

9. Adding $-x$ and x gives you 0, so if you add both tiles, you don't change the value of the polynomial.
10. $(x - 2)(x + 1)$
11. $(x - 3)(x + 1)$
12. $(x - 1)(x - 4)$
13. $(x - 2)(x - 5)$
14. $(x - 1)(x - 1)$
15. $(x - 1)(x - 5)$
16. $(x - 1)(x + 6)$
17. $(x - 1)(x + 4)$
18. $(x - 3)(x + 2)$
19. $(x - 2)(x + 5)$
20. $(x - 4)(x + 2)$
21. $(x - 3)(x + 4)$

Teacher to Teacher

I have found that using algebra tiles really helps students achieve success with algebraic concepts. Since students are already familiar with the area of rectangles, connecting the factors of a trinomial with the dimensions of a rectangle provides a comfortable introduction to factoring trinomials.

Make sure that everyone knows what to call each tile so that students can discuss the ideas constructively in class and with each other outside of class.

Vicki Petty
Murfreesboro, TN

7-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Factor quadratic trinomials of the form $x^2 + bx + c$.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

1. Which pair of factors of 8 has a sum of 9? 1 and 8
2. Which pair of factors of 30 has a sum of −17? −2 and −15

Multiply.

3. $(x + 2)(x + 3)$ $x^2 + 5x + 6$
4. $(r + 5)(r - 9)$ $r^2 - 4r - 45$

Also available online

Math Humor

Q: What did the two binomials say when they were multiplied?

A: Curses! FOILed again!

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Learn It Online
State Resources Online

COMMON CORE

7-3 Factoring $x^2 + bx + c$

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objective
Factor quadratic trinomials of the form $x^2 + bx + c$.

Why learn this?
Factoring polynomials will help you find the dimensions of rectangular shapes, such as a fountain. (See Exercise 71.)

Previously, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binomials.

Notice that when you multiply $(x + 2)(x + 5)$, the constant term in the trinomial is the product of the constants in the binomials.

$$(x + 2)(x + 5) = x^2 + 7x + 10$$

You can use this fact to factor a trinomial into its binomial factors. Look for two numbers that are factors of the constant term in the trinomial. Write two binomials with those numbers, and then multiply to see if you are correct.

EXAMPLE 1 **Factoring Trinomials by Guess and Check**

Factor $x^2 + 19x + 60$ by guess and check.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

Remember!
When you multiply two binomials, multiply:
First terms
Outer terms
Inner terms
Last terms

The constant term in the trinomial is 60.

$(x + 1)(x + 60) = x^2 + 61x + 60$ ✗
$(x + 2)(x + 30) = x^2 + 32x + 60$ ✗
$(x + 3)(x + 20) = x^2 + 23x + 60$ ✗
$(x + 4)(x + 15) = x^2 + 19x + 60$ ✓

Try factors of 60 for the constant terms in the binomials.

The factors of $x^2 + 19x + 60$ are $(x + 4)$ and $(x + 15)$.

$x^2 + 19x + 60 = (x + 4)(x + 15)$

CHECK IT OUT! **Factor each trinomial by guess and check.**

1a. $x^2 + 10x + 24$ $(x + 4)(x + 6)$

1b. $x^2 + 7x + 12$ $(x + 4)(x + 3)$

Travel-Shots / Alamy

1 Introduce

EXPLORATION

Factoring $x^2 + bx + c$

Look for patterns as you multiply binomials. The patterns that you discover will help you factor trinomials.

1. Complete the table by multiplying the binomial factors. Be sure to write the products as trinomials in the form $x^2 + bx + c$.

Factors	Product
$(x + 2)(x + 4)$	$x^2 + 6x + 8$
$(x + 5)(x + 3)$	$x^2 + 8x + 15$
$(x + 3)(x - 7)$	$x^2 - 4x - 21$
$(x - 5)(x + 2)$	$x^2 - 3x - 10$
$(x - 5)(x - 9)$	$x^2 - 14x + 45$

2. Look at the constant terms of the trinomials. How are these related to the constant terms of the binomial factors?
3. Look at the coefficients of the middle terms of the trinomials. How are these related to the constant terms of the binomial factors?

THINK AND DISCUSS

4. **Discuss** what you know about the constant terms of the binomial factors of $x^2 + 10x + 24$.
5. **Explain** how you know that $(x + 12)(x + 2)$ is not a correct factorization of $x^2 + 10x + 24$.

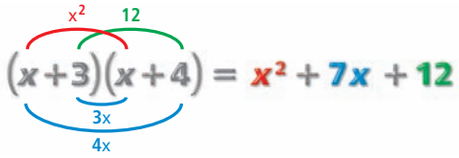
2. They are the products of the constant terms.

Motivate

Ask students when they have seen inverse operations—for example, addition and subtraction, multiplication and division, squaring a number and finding its square root. Tell students that in this lesson, they will learn how to undo binomial multiplication by factoring.

Explorations and answers are provided in your online resources.

The guess and check method is usually not the most efficient method of factoring a trinomial. Look at the product of $(x + 3)$ and $(x + 4)$.



The coefficient of the middle term is the sum of 3 and 4. The third term is the product of 3 and 4.

Know it! Note — Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b. If no such integers exist, the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9. Factors of 18 | Sum 1 and 18 | 19 ✗ 2 and 9 | 11 ✗ 3 and 6 | 9 ✓ $x^2 + 9x + 18 = (x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive, and when b is negative, the factors are negative.

EXAMPLE 2 Factoring $x^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $x^2 + 6x + 8$

$(x + \square)(x + \square)$ *b = 6 and c = 8; look for factors of 8 whose sum is 6.*

Factors of 8	Sum	
1 and 8	9	✗
2 and 4	6	✓

The factors needed are 2 and 4.

$(x + 2)(x + 4)$

Check $(x + 2)(x + 4) = x^2 + 4x + 2x + 8$ *Use the FOIL method.*

$= x^2 + 6x + 8$ ✓ *The product is the original polynomial.*

B $x^2 + 5x + 6$

$(x + \square)(x + \square)$ *b = 5 and c = 6; look for factors of 6 whose sum is 5.*

Factors of 6	Sum	
1 and 6	7	✗
2 and 3	5	✓

The factors needed are 2 and 3.

$(x + 2)(x + 3)$

Check $(x + 2)(x + 3) = x^2 + 3x + 2x + 6$ *Use the FOIL method.*

$= x^2 + 5x + 6$ ✓ *The product is the original polynomial.*

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Additional Examples

Example 1

Factor $x^2 + 15x + 36$.

$(x + 3)(x + 12)$

Example 2

Factor each trinomial. Check your answer.

A. $x^2 + 6x + 5$ $(x + 1)(x + 5)$

B. $x^2 + 6x + 9$ $(x + 3)(x + 3)$

C. $x^2 - 8x + 15$ $(x - 3)(x - 5)$

Also available online

Mathematical Practices — Questioning Strategies

EXAMPLE 1

- For which number do you try different pairs of factors?

EXAMPLE 2

- What does the sign of c tell you about the factors?
- What does the sign of b tell you about the factors?

Teaching Tip **Visual** Suggest that students write down what the product of the two constants in the binomials should be and what their sum should be. They can write "P": and "S": next to the trinomial as a reminder to themselves.

$x^2 + 13x + 36$ P: 36 S: 13

2 Teach

Guided Instruction

Review the FOIL method before factoring any trinomials. Remind students that the FOIL method is an application of the Distributive Property. Have students examine the product of two binomials, such as $(x + 2)(x + 3) = x^2 + 5x + 6$, and discuss where each term in the trinomial came from. Lead them to see that $5x$ came from the sum of like terms $2x$ and $3x$ and that 6 came from the product of constants 2 and 3.

Reaching All Learners

Through Graphic Organizers

Have students make a chart like the one below, filling in their own examples. Then have students factor each example.

b	c	Example	Sign of Factors	Factors
+	+	$x^2 + 5x + 4$	Both positive	$(x + 1)(x + 4)$
−	+	$n^2 - 6n + 8$	Both negative	$(n - 2)(n - 4)$
−	−	$t^2 - t - 6$	Factor with greater abs. val. is negative.	$(t - 3)(t + 2)$
+	−	$q^2 + 3q - 18$	Factor with greater abs. val. is positive.	$(q + 6)(q - 3)$

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Additional Examples

Example 3

Factor each trinomial.

A. $x^2 + x - 20$ $(x + 5)(x - 4)$

B. $x^2 - 3x - 18$ $(x - 6)(x + 3)$

Also available online

Questioning Strategies

EXAMPLE 3

- How are these problems like those in **Example 2**?
- How do you know which factor will be positive and which will be negative?

Teaching Tip **Inclusion** As students begin to factor trinomials in which c is negative, remind them that the absolute value of a number is the number's distance from 0 on a number line.

Factor each trinomial. Check your answer.

C $x^2 - 10x + 16$

$(x + \square)(x + \square)$ *$b = -10$ and $c = 16$; look for factors of 16 whose sum is -10.*

Factors of 16	Sum	
-1 and -16	-17	✗
-2 and -8	-10	✓

The factors needed are -2 and -8.

$(x - 2)(x - 8)$

Check $(x - 2)(x - 8) = x^2 - 8x - 2x + 16$ *Use the FOIL method.*

$= x^2 - 10x + 16$ ✓ *The product is the original polynomial.*

CHECK IT OUT! **Factor each trinomial. Check your answer.**

2a. $x^2 + 8x + 12$ $(x + 6)(x + 2)$ **2b.** $x^2 - 5x + 6$ $(x - 2)(x - 3)$

2c. $x^2 + 13x + 42$ $(x + 6)(x + 7)$ **2d.** $x^2 - 13x + 40$ $(x - 8)(x - 5)$

When c is negative, its factors have opposite signs. The sign of b tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as b.

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor each trinomial.

A $x^2 + 7x - 18$

$(x + \square)(x + \square)$ *$b = 7$ and $c = -18$; look for factors of -18 whose sum is 7. The factor with the greater absolute value is positive.*

Factors of -18	Sum	
-1 and 18	17	✗
-2 and 9	7	✓

The factors needed are -2 and 9.

$(x - 2)(x + 9)$

Helpful Hint

If you have trouble remembering the rules for which factor is positive and which is negative, you can try all the factor pairs and check their sums.

B $x^2 - 5x - 24$

$(x + \square)(x + \square)$ *$b = -5$ and $c = -24$; look for factors of -24 whose sum is -5. The factor with the greater absolute value is negative.*

Factors of -24	Sum	
1 and -24	-23	✗
2 and -12	-10	✗
3 and -8	-5	✓

The factors needed are 3 and -8.

$(x + 3)(x - 8)$

CHECK IT OUT! **Factor each trinomial. Check your answer.** $(x - 10)(x + 2)$

3a. $x^2 + 2x - 15$ **3b.** $x^2 - 6x + 8$ **3c.** $x^2 - 8x - 20$

3a. $(x + 5)(x - 3)$

3b. $(x - 4)(x - 2)$

Answer to *Check It Out*

4.

n	$n^2 - 7n + 10$	$(n - 5)(n - 2)$
0	$0^2 - 7(0) + 10 = 10$	$(0 - 5)(0 - 2) = 10$
1	$1^2 - 7(1) + 10 = 4$	$(1 - 5)(1 - 2) = 4$
2	$2^2 - 7(2) + 10 = 0$	$(2 - 5)(2 - 2) = 0$
3	$3^2 - 7(3) + 10 = -2$	$(3 - 5)(3 - 2) = -2$
4	$4^2 - 7(4) + 10 = -2$	$(4 - 5)(4 - 2) = -2$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

EXAMPLE 4 Evaluating Polynomials

Factor $n^2 + 11n + 24$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

$n^2 + 11n + 24$

$(n + \square)(n + \square)$ — *b = 11 and c = 24; look for factors of 24 whose sum is 11.*

Factors of 24	Sum	
1 and 24	25	✗
2 and 12	14	✗
3 and 8	11	✓

The factors needed are 3 and 8.

$(n + 3)(n + 8)$

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3,$ and 4.

n	$n^2 + 11n + 24$
0	$0^2 + 11(0) + 24 = 24$
1	$1^2 + 11(1) + 24 = 36$
2	$2^2 + 11(2) + 24 = 50$
3	$3^2 + 11(3) + 24 = 66$
4	$4^2 + 11(4) + 24 = 84$

n	$(n + 3)(n + 8)$
0	$(0 + 3)(0 + 8) = 24$
1	$(1 + 3)(1 + 8) = 36$
2	$(2 + 3)(2 + 8) = 50$
3	$(3 + 3)(3 + 8) = 66$
4	$(4 + 3)(4 + 8) = 84$

The original polynomial and the factored form have the same value for the given values of n.

CHECK IT OUT! **4.** Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

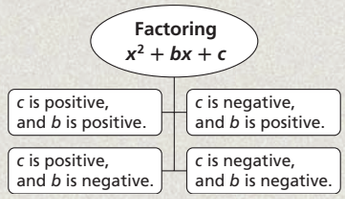
$(n - 5)(n - 2)$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain in your own words how to factor $x^2 + 9x + 14$. Show how to check your answer.
2. Explain how you can determine the signs of the factors of c when factoring a trinomial of the form $x^2 + bx + c$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a trinomial with the given properties and factor it.

Know it! Note



COMMON ERROR ALERT

When factoring a trinomial with a negative c, students may write a minus sign in the binomial factor with the greater constant term. Remind students that the binomial factor with the greater constant term should have the same sign as b.

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Additional Examples

Example 4

Factor $y^2 + 10y + 21$. Show that the original polynomial and the factored form have the same value for $y = 0, 1, 2, 3,$ and 4.

$(y + 7)(y + 3)$

y	$y^2 + 10y + 21$
0	$0^2 + 10(0) + 21 = 21$
1	$1^2 + 10(1) + 21 = 32$
2	$2^2 + 10(2) + 21 = 45$
3	$3^2 + 10(3) + 21 = 60$
4	$4^2 + 10(4) + 21 = 77$

y	$(y + 7)(y + 3)$
0	$(0 + 7)(0 + 3) = 21$
1	$(1 + 7)(1 + 3) = 32$
2	$(2 + 7)(2 + 3) = 45$
3	$(3 + 7)(3 + 3) = 60$
4	$(4 + 7)(4 + 3) = 77$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- How would you know if the trinomial was not factored correctly?

3 Close

Summarize

Review with students how to factor trinomials of all four types: two cases when c is positive and two cases when c is negative.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. Find the 2 factors of 14 that have a sum of 9: 2 and 7. Then use these numbers as the constants in the factors: $(x + 2)(x + 7)$.
2. For $x^2 + bx + c = (x + m)(x + n)$, if $c > 0$ and $b > 0$, $m > 0$ and $n > 0$. If $c > 0$ and $b < 0$, $m < 0$ and $n < 0$. If $c < 0$ and $b > 0$, the greater of m and n is positive and the lesser is negative. If $c < 0$ and $b < 0$, the greater of m and n is negative and the lesser is positive.
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**

Basic 17–25, 52, 55, 60, 70
Average 17–25, 60, 70, 71, 77
Advanced 17–25, 70, 71, 77, 83

If you finished Examples **1–4**

Basic 17–36, 51–66, 73–76
Average 17–50, 53, 62–76, 77–83 odd
Advanced 17–32, 37, 51–57 odd, 62–85

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 22, 30, 32, 53, 60

Answers

4. $(x + 1)(x + 3)$
5. $(x + 2)(x + 8)$
6. $(x + 4)(x + 11)$
7. $(x - 1)(x - 6)$
8. $(x - 2)(x - 7)$
9. $(x - 3)(x - 8)$
10. $(x - 7)(x + 1)$
11. $(x + 9)(x - 3)$
12. $(x + 6)(x - 5)$
13. $(x - 2)(x + 1)$
14. $(x - 6)(x + 3)$
15. $(x - 9)(x + 5)$
16. $(n + 7)(n - 1)$; for table see Additional Answers.
20. $(x + 1)(x + 11)$
21. $(x + 2)(x + 14)$

22–32, 38–50. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 52–53, 60–62, 71

Reason abstractly and quantitatively. Exercises 63–66, 83–85

Construct viable arguments and critique the reasoning of others. Exercises 16, 32, 37, 50, 72

Model with mathematics. Exercises 54–56

Attend to precision. Exercise 51

Look for and make use of structure. Exercises 57–59, 67–70

Learn It Online
State Resources Online

7-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Factor each trinomial by guess and check.

1. $x^2 + 13x + 36$ $(x + 4)(x + 9)$
2. $x^2 + 11x + 24$ $(x + 3)(x + 8)$
3. $x^2 + 14x + 40$ $(x + 4)(x + 10)$

Factor each trinomial. Check your answer.

SEE EXAMPLE 2

4. $x^2 + 4x + 3$
5. $x^2 + 10x + 16$
6. $x^2 + 15x + 44$
7. $x^2 - 7x + 6$
8. $x^2 - 9x + 14$
9. $x^2 - 11x + 24$

SEE EXAMPLE 3

10. $x^2 - 6x - 7$
11. $x^2 + 6x - 27$
12. $x^2 + x - 30$
13. $x^2 - x - 2$
14. $x^2 - 3x - 18$
15. $x^2 - 4x - 45$

SEE EXAMPLE 4

16. Factor $n^2 + 6n - 7$. Show that the original polynomial and the factored form have the same value for $n = 0$, 1, 2, 3, and 4.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–25	2
26–31	3
32	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Factor each trinomial by guess and check.

17. $x^2 + 13x + 30$ $(x + 3)(x + 10)$
18. $x^2 + 11x + 28$ $(x + 4)(x + 7)$
19. $x^2 + 16x + 48$ $(x + 4)(x + 12)$

Factor each trinomial. Check your answer.

20. $x^2 + 12x + 11$
21. $x^2 + 16x + 28$
22. $x^2 + 15x + 36$
23. $x^2 - 6x + 5$
24. $x^2 - 9x + 18$
25. $x^2 - 12x + 32$
26. $x^2 + x - 12$
27. $x^2 + 4x - 21$
28. $x^2 + 9x - 36$
29. $x^2 - 12x - 13$
30. $x^2 - 10x - 24$
31. $x^2 - 2x - 35$
32. Factor $n^2 - 12n - 45$. Show that the original polynomial and the factored form have the same value for $n = 0$, 1, 2, 3, and 4.

Match each trinomial with its correct factorization.

33. $x^2 + 3x - 10$ **C**
34. $x^2 - 7x + 10$ **A**
35. $x^2 - 9x - 10$ **D**
36. $x^2 + 11x + 10$ **B**

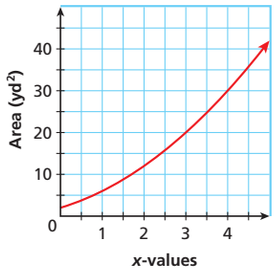
A. $(x - 2)(x - 5)$
B. $(x + 1)(x + 10)$
C. $(x - 2)(x + 5)$
D. $(x + 1)(x - 10)$

37. **Write About It** Compare multiplying binomials with factoring polynomials into binomial factors. They are inverse operations.

Factor each trinomial. Check your answer.

38. $x^2 + x - 20$
39. $x^2 - 11x + 18$
40. $x^2 - 4x - 21$
41. $x^2 + 10x + 9$
42. $x^2 - 12x + 32$
43. $x^2 + 13x + 42$
44. $x^2 - 7x + 12$
45. $x^2 + 11x + 18$
46. $x^2 - 6x - 27$
47. $x^2 + 5x - 24$
48. $x^2 - 10x + 21$
49. $x^2 + 4x - 45$
50. Factor $n^2 + 11n + 28$. Show that the original polynomial and the factored form have the same value for $n = 0$, 1, 2, 3, and 4.

51. Estimation The graph shows the areas of rectangles with dimensions $(x + 1)$ yards and $(x + 2)$ yards. Estimate the value of x for a rectangle with area 9 square yards. **approximately 1.5**



52. Geometry The area of a rectangle in square feet can be represented by $x^2 + 8x + 12$. The length is $(x + 6)$ ft. What is the width of the rectangle? **$(x + 2)$ ft**

53. Remodeling A homeowner wants to enlarge a rectangular closet that has an area of $(x^2 + 3x + 2)$ ft^2. The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft^2 with a length of $(x + 3)$ ft.

a. Find the dimensions of the closet before construction.

b. Find the dimensions of the closet after construction.

c. By how many feet will the length and width increase after construction?
The length will increase by 1 ft. The width will increase by 4 ft.

LINK

Art

The Dutch painter Theo van Doesburg (1883–1931) is most famous for his paintings composed of lines and rectangles, such as the one shown above.

Art Write the polynomial modeled and then factor.

54.

$x^2 + 5x + 6$; $(x + 3)(x + 2)$

55.

$x^2 + 6x + 8$; $(x + 4)(x + 2)$

56.

$x^2 + 2x - 8$; $(x + 4)(x - 2)$

Copy and complete the table.

	$x^2 + bx + c$	Sign of c	Binomial Factors	Signs of Numbers in Binomials
	$x^2 + 4x + 3$	Positive	$(x + 1)(x + 3)$	Both positive
57.	$x^2 - 4x + 3$	**Pos.**	$(x - 1)(x - 3)$	**Both neg.**
58.	$x^2 + 2x - 3$	**Neg.**	$(x - 1)(x + 3)$	**Neg.; pos.**
59.	$x^2 - 2x - 3$	**Neg.**	$(x + 1)(x - 3)$	**Pos.; neg.**

60. Geometry A rectangle has area $x^2 + 6x + 8$. The length is $x + 4$. Find the width of the rectangle. Could the rectangle be a square? Explain why or why not.
$x + 2$; no; Possible answer: $2 \neq 4$ so $x + 2 \neq x + 4$

MULTI-STEP TEST PREP

61. The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in feet per second, a is acceleration in feet per second squared, and t is time in seconds.

a. Janna has two toy race cars on a track. One starts with a velocity of 0 ft/s and accelerates at 2 ft/s^2. Write an equation for the distance the car travels in time t. **$d = t^2$**

b. The second car travels at a constant speed of 4 ft/s. Write an equation for the distance the second car travels in time t. (*Hint:* When speed is constant, the acceleration is 0 ft/s^2.) **$d = 4t$**

c. By setting the equations equal to each other you can determine when the cars have traveled the same distance: $t^2 = 4t$. This can be written as $t^2 - 4t = 0$. Factor the left side of the equation. **$t(t - 4)$**

(cl), Composition 17, 1919 (oil on canvas) by Doesburg, Theo van (1883–1931) ©Haags Gemeentemuseum, The Hague, Netherlands/The Bridgeman Art Library Nationality/copyright status: Dutch/out of copyright; (b), HMH

COMMON ERROR ALERT

Students might assign the larger value to length in part **b** of **Exercise 53.** This would yield a remodeled length of $x + 5$. Students may then carry the error over into part **c,** answering that length changed by 3 and width changed by 2. Tell students to read the problem carefully. Draw their attention to the last line. The problem clearly states that the new length is $x + 3$, so the new *width* is $x + 5$.

Answers

53a. length: $(x + 2)$ ft; width: $(x + 1)$ ft

b. length: $(x + 3)$ ft; width: $(x + 5)$ ft

MULTI-STEP TEST PREP **Exercise 61** involves using the equation $d = vt + \frac{1}{2}at^2$ to calculate when two moving objects will be at the same location. This exercise prepares students for the Multi-Step Test Prep.

PRACTICE A

PRACTICE C

PRACTICE B

Factor each trinomial.

1. $x^2 + 7x + 10$ — $(x + 2)(x + 5)$
2. $x^2 + 9x + 8$ — $(x + 1)(x + 8)$
3. $x^2 + 13x + 36$ — $(x + 4)(x + 9)$
4. $x^2 + 9x + 14$ — $(x + 7)(x + 2)$
5. $x^2 + 7x + 12$ — $(x + 3)(x + 4)$
6. $x^2 + 9x + 18$ — $(x + 6)(x + 3)$
7. $x^2 - 9x + 18$ — $(x - 6)(x - 3)$
8. $x^2 - 5x + 4$ — $(x - 4)(x - 1)$
9. $x^2 - 9x + 20$ — $(x - 5)(x - 4)$
10. $x^2 - 12x + 20$ — $(x - 2)(x - 10)$
11. $x^2 - 11x + 18$ — $(x - 9)(x - 2)$
12. $x^2 - 12x + 32$ — $(x - 8)(x - 4)$
13. $x^2 + 7x - 18$ — $(x + 9)(x - 2)$
14. $x^2 + 10x - 24$ — $(x + 12)(x - 2)$
15. $x^2 + 2x - 3$ — $(x + 3)(x - 1)$
16. $x^2 + 2x - 15$ — $(x + 5)(x - 3)$
17. $x^2 + 5x - 6$ — $(x + 6)(x - 1)$
18. $x^2 + 5x - 24$ — $(x + 8)(x - 3)$
19. $x^2 - 5x - 6$ — $(x + 1)(x - 6)$
20. $x^2 - 2x - 35$ — $(x + 5)(x - 7)$
21. $x^2 - 7x - 30$ — $(x + 3)(x - 10)$
22. $x^2 - x - 56$ — $(x + 7)(x - 8)$
23. $x^2 - 2x - 8$ — $(x + 2)(x - 4)$
24. $x^2 - x - 20$ — $(x + 4)(x - 5)$

25. Factor $n^2 + 5n - 24$. Show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3$, and 4. **$(n + 8)(n - 3)$**

n	$n^2 + 5n - 24$
0	$0^2 + 5(0) - 24 = -24$
1	$1^2 + 5(1) - 24 = -18$
2	$2^2 + 5(2) - 24 = -10$
3	$3^2 + 5(3) - 24 = 0$
4	$4^2 + 5(4) - 24 = 12$

n	$(n + 8)(n - 3)$
0	$(0 + 8)(0 - 3) = -24$
1	$(1 + 8)(1 - 3) = -18$
2	$(2 + 8)(2 - 3) = -10$
3	$(3 + 8)(3 - 3) = 0$
4	$(4 + 8)(4 - 3) = 12$

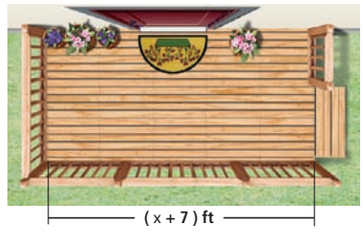
TEST PREP DOCTOR Some students will be drawn to choice **A** for **Exercise 73** because $-4 + -6 = -10$. Remind them that the product of two negative numbers is positive.

Teaching Tip **Critical Thinking** Have students substitute a single variable for the common binomial expression in **Exercises 80** and **82**. Then, when they have factored the trinomial, they can substitute the binomial expression back in for the single variable.

Answers

64. The correct factorization is $(x - 1)(x + 2)$.
65. The correct factorization is $(x - 4)(x + 1)$.
66. Many trinomials cannot be factored.
77. $(x^2 + 9)(x^2 + 9)$
78. $(y^2 - 8)(y^2 + 3)$
79. $(d^2 + 21)(d^2 + 1)$
80. $(u + v + 3)(u + v - 1)$
81. $(de - 5)(de + 4)$
82. $(m - n - 9)(m - n + 5)$

62. Construction The length of a rectangular porch is $(x + 7)$ ft. The area of the porch is $(x^2 + 9x + 14)$ ft^2. Find the width of the porch. $(x + 2)$ ft



Tell whether each statement is true or false. If false, explain.

63. The third term in a factorable trinomial is equal to the product of the constants in its binomial factors. **true**

64. The constants in the binomial factors of $x^2 + x - 2$ are both negative. **false**

65. The correct factorization of $x^2 - 3x - 4$ is $(x + 4)(x - 1)$. **false**

66. All trinomials of the form $x^2 + bx + c$ can be factored. **false**

Fill in the missing part of each factorization.

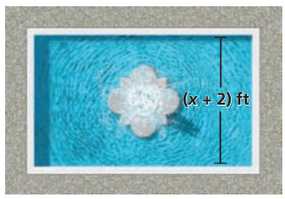
67. $x^2 - 6x + 8 = (x - 2)(x - \boxed{4})$

68. $x^2 - 2x - 8 = (x + 2)(x - \boxed{4})$

69. $x^2 + 2x - 8 = (x - 2)(x + \boxed{4})$

70. $x^2 + 6x + 8 = (x + 2)(x + \boxed{4})$

71. Construction The area of a rectangular fountain is $(x^2 + 12x + 20)$ ft^2. The width is $(x + 2)$ ft.



a. Find the length of the fountain. $(x + 10)$ ft

b. A 2-foot walkway is built around the fountain. Find the dimensions of the outside border of the walkway. $\ell = (x + 14)$ ft; $w = (x + 6)$ ft

c. Find the total area covered by the fountain and walkway. $A = (x^2 + 20x + 84)$ ft^2

72. Critical Thinking Find all possible values of b so that $x^2 + bx + 6$ can be factored into binomial factors. **7, 5, −7, −5**

TEST PREP

73. Which is the correct factorization of $x^2 - 10x - 24$?

Ⓐ $(x - 4)(x - 6)$ Ⓒ $(x - 2)(x + 12)$
Ⓑ $(x + 4)(x - 6)$ Ⓓ $(x + 2)(x - 12)$

74. Which value of b would make $x^2 + bx - 20$ factorable?

Ⓕ 9 Ⓖ 12 Ⓗ 19 Ⓙ 21

75. Which value of b would NOT make $x^2 + bx - 36$ factorable?

Ⓐ 5 Ⓑ 9 Ⓒ 15 Ⓓ 16

76. Short Response What are the factors of $x^2 + 2x - 24$? Show and explain each step of factoring the polynomial. $(x + 6)(x - 4)$

CHALLENGE AND EXTEND

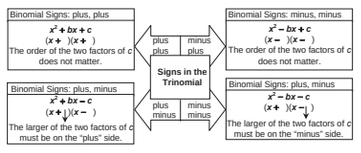
Factor each expression.

77. $x^4 + 18x^2 + 81$ **78.** $y^4 - 5y^2 - 24$ **79.** $d^4 + 22d^2 + 21$

80. $(u + v)^2 + 2(u + v) - 3$ **81.** $(de)^2 - (de) - 20$ **82.** $(m - n)^2 - 4(m - n) - 45$

READING STRATEGIES

To factor trinomials of the form $x^2 + bx + c$ into two binomials, it is important to pay attention to the signs. Although you must figure out which factors of c will work, you can use the graphic organizer below to help you sort out the signs.



Fill in the signs of the two binomials that will form the correct factors of the given trinomial.

1. $x^2 + 9x + 18$ → $(x + 6)(x + 3)$
2. $x^2 - 3x - 10$ → $(x + 2)(x - 5)$
3. $x^2 + 13x + 36$ → $(x + 9)(x + 4)$
4. $x^2 - 7x + 12$ → $(x - 3)(x - 4)$
5. $x^2 + 3x - 4$ → $(x + 4)(x - 1)$
6. $x^2 - 9x + 14$ → $(x - 7)(x - 2)$
7. $x^2 + 5x - 50$ → $(x + 10)(x - 5)$
8. $x^2 - x - 42$ → $(x + 6)(x - 7)$
9. $x^2 + 6x + 9$ → $(x + 3)(x + 3)$

Factor each trinomial into two binomials.

10. $x^2 - 10x + 21$ → $(x - 3)(x - 7)$
11. $x^2 + 5x - 24$ → $(x + 8)(x - 3)$
12. $x^2 + 11x + 30$ → $(x + 6)(x + 5)$

RETEACH

When factoring $x^2 + bx + c$:

If c is positive	and b is positive	both factors are positive.
	and b is negative	both factors are negative.

Factor $x^2 + 7x + 10$. Check your answer.

$x^2 + 7x + 10$

Need factors of **10** that sum to **7**.

Factors of 10	Sum
1 and 10	11 ×
2 and 5	7 ✓

$(x + 2)(x + 5)$

Check:

$(x + 2)(x + 5) = x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$ ✓

Factor $x^2 - 9x + 18$. Check your answer.

$x^2 - 9x + 18$

Need factors of **18** that sum to **−9**.

Factors of 18	Sum
−1 and −18	−19 ×
−2 and −9	−11 ×
−3 and −6	−9 ✓

$(x - 3)(x - 6)$

Check:

$(x - 3)(x - 6) = x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$ ✓

Factor the trinomial by filling in the blanks below.

1. $x^2 + 10x + 16$

Need factors of 16 that sum to 10

Factors	Sum
1 and 16	17
2 and 8	10
4 and 4	8

$(x + 2)(x + 8)$

2. $x^2 - 9x + 20$

Need factors of 20 that sum to −9

Factors	Sum
−1 and −20	−21
−2 and −10	−12
−4 and −5	−9

$(x - 4)(x - 5)$

Factor each trinomial.

3. $x^2 + 13x + 12$ → $(x + 12)(x + 1)$
4. $x^2 + 15x + 50$ → $(x + 10)(x + 5)$
5. $x^2 - 13x + 36$ → $(x - 9)(x - 4)$

83. Find all possible values of b such that, when $x^2 + bx + 28$ is factored, both constants in the binomials are positive. **16; 11; 29**

84. Find all possible values of b such that, when $x^2 + bx + 32$ is factored, both constants in the binomials are negative. **−33; −18; −12**

85. The area of Beth's rectangular garden is $(x^2 + 13x + 42)$ ft^2. The width is $(x + 6)$ ft.

Item	Cost
Fertilizer	0.28 ($/ft^2)
Fencing	2.00 ($/ft)

a. What is the length of the garden? **$(x + 7)$ ft**

b. Find the perimeter in terms of x. **$(4x + 26)$ ft**

c. Find the cost to fence the garden when x is 5. **$92.00**

d. Find the cost of fertilizer when x is 5. **$36.96**

e. Find the total cost to fence and fertilize Beth's garden when x is 5. **$128.96**

Career Path

Learn It Online
Career Resources Online

Jessica Rubino
Environmental Sciences major

Q: What math classes did you take in high school?
A: Algebra 1, Algebra 2, and Geometry

Q: What college math classes have you taken?
A: I took several computer modeling and programming classes as well as Statistics and Probability.

Q: How is math used in some of your projects?
A: Computer applications help me analyze data collected from a local waste disposal site. I used my mathematical knowledge to make recommendations on how to preserve surrounding water supplies.

Q: What plans do you have for the future?
A: I enjoy my studies in the area of water pollution. I would also like to research more efficient uses of natural energy resources.

Janine Wiedel Photolibrary/Alamy

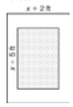
PROBLEM SOLVING

Write the correct answer.

1. A plot of land is rectangular and has an area of $x^2 - 5x - 24$ m^2. The length is $x + 3$ m. Find the width of the plot.
$x - 8$ m

3. The area of a poster board is $x^2 + 3x - 10$ inches. The width is $x - 2$ inches.
   a. Write an expression for the length of the poster board.
   $x + 5$
   b. Find the dimensions of the poster board when $x = 14$.
   12 in. by 19 in.
   c. Write a polynomial for the area of the poster board if one inch is removed from each side.
   $x^2 + x - 12$

2. An antique Persian carpet has an area of $(x^2 + x - 20)$ ft^2 and a length of $(x + 5)$ feet. The rug is displayed on a wall in a museum. The wall has a width of $(x + 2)$ feet and an area of $(x^2 + 17x + 30)$ ft^2. Write expressions for the length and width of both the rug and wall. Then find the dimensions of the rug and the wall if $x = 20$ feet.



rug: $(x + 5)(x - 4)$;
wall: $(x + 2)(x + 15)$;
rug: 16 ft by 25 ft,
wall: 22 ft by 35 ft

The figure shows the plans for an addition on the back of a house. Use the figure to answer questions 4–6. Select the best answer.

4. The area of the addition is $(x^2 + 10x - 200)$ ft^2. What is its length?
   A $(x - 20)$ feet
   B $(x - 2)$ feet
   C $(x + 2)$ feet
   D $(x + 20)$ feet

5. What is the area of the original house?
   F $(x^2 - 10x - 200)$ ft^2
   G $(x^2 + 8x - 20)$ ft^2
   H $(x^2 + 12x + 200)$ ft^2
   J $(x^2 + 30x + 200)$ ft^2

6. The homeowners decide to extend the addition. The area with the addition is now $(x^2 + 12x - 160)$ ft^2. By how many feet was the addition extended?
   A 1 foot C 3 feet
   B 2 feet D 4 feet

CHALLENGE

One day, Denise's mathematics teacher, Ms. Dolenze, asked her to factor $6x^2 + 5x + 1$. After puzzling over the problem for a while, Denise told Ms. Dolenze that the expression could not be factored. Amazed, Ms. Dolenze asked Denise why. "I could not find two binomials in x with integer coefficients whose product is $6x^2 + 5x + 1$," replied Denise. Then Ms. Dolenze quickly responded, "Why should that stop you?"

In the exercises that follow, you will see how to push back the boundaries of factoring.

In Exercise 1–4, answer each question in order to help Denise factor $6x^2 + 5x + 1$.

1. By what property of real numbers, can you justify $6x^2 + 5x + 1 = 6\left(x^2 + \frac{5}{6}x + \frac{1}{6}\right)$?
Distributive Property

2. Show that $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$ and that $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.
$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1 \times 1}{2 \times 3} = \frac{1}{6} \cdot \frac{1}{2} + \frac{1}{3} = \frac{1}{2} \times \frac{3}{3} + \frac{1}{3} \times \frac{2}{2} = \frac{3+2}{2 \times 3} = \frac{5}{6}$

3. Show that $6\left(x + \frac{1}{2}\right)\left(x + \frac{1}{3}\right) = 6x^2 + 5x + 1$.
$6\left(x + \frac{1}{2}\right)\left(x + \frac{1}{3}\right) = 2\left(x + \frac{1}{2}\right) \times 3\left(x + \frac{1}{3}\right) = (2x + 1)(3x + 1)$
$= 6x^2 + 3x + 2x + 1 = 6x^2 + 5x + 1$

4. Is the product in Exercise 3 a factorization of $6x^2 + 5x + 1$? Is it necessary to find integers in order to factor a quadratic trinomial? Explain your response.
Yes; the given polynomial expression is written as a product of polynomials of lower degree. There is no requirement that the coefficients be integers.

Factor each quadratic trinomial as a number times a product of two linear binomials in x, each having rational numbers among the coefficients. Use Exercises 1–3 above as a model.

5. $15x^2 + 8x + 1$
$15\left(x + \frac{1}{5}\right)\left(x + \frac{1}{3}\right)$

6. $28x^2 + 11x + 1$
$28\left(x + \frac{1}{7}\right)\left(x + \frac{1}{4}\right)$

7. $6x^2 - 23x + 20$
$6\left(x - \frac{4}{3}\right)\left(x - \frac{5}{2}\right)$

8. $12x^2 + x - 35$
$12\left(x - \frac{5}{3}\right)\left(x + \frac{7}{4}\right)$

Journal

Have students compare factoring a trinomial when c is negative and b is positive with factoring a trinomial when c is negative and b is negative.

ALTERNATIVE ASSESSMENT

Have students write four trinomials by multiplying four pairs of binomials. In two of the trinomials, c should be positive. Have students trade papers and factor the trinomials they received.

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7-3 Lesson Quiz

Factor each trinomial.

1. $x^2 - 11x + 30$
$(x - 5)(x - 6)$

2. $x^2 + 10x + 9$
$(x + 1)(x + 9)$

3. $x^2 - 6x - 27$
$(x - 9)(x + 3)$

4. $x^2 + 14x - 32$
$(x + 16)(x - 2)$

5. Factor $n^2 + n - 6$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3$, and 4. $(n + 3)(n - 2)$

n	$n^2 + n - 6$
0	$0^2 + 0 - 6 = -6$
1	$1^2 + 1 - 6 = -4$
2	$2^2 + 2 - 6 = 0$
3	$3^2 + 3 - 6 = 6$
4	$4^2 + 4 - 6 = 14$

n	$(n + 3)(n - 2)$
0	$(0 + 3)(0 - 2) = -6$
1	$(1 + 3)(1 - 2) = -4$
2	$(2 + 3)(2 - 2) = 0$
3	$(3 + 3)(3 - 2) = 6$
4	$(4 + 3)(4 - 2) = 14$

Also available online

7-4 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objective: Factor quadratic trinomials of the form $ax^2 + bx + c$.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find each product.

1. $(x - 2)(2x + 7)$ $2x^2 + 3x - 14$
2. $(3y + 4)(2y + 9)$ $6y^2 + 35y + 36$
3. $(3n - 5)(n - 7)$ $3n^2 - 26n + 35$

Factor each trinomial.

4. $x^2 + 4x - 32$ $(x - 4)(x + 8)$
5. $z^2 + 15z + 36$ $(z + 3)(z + 12)$
6. $h^2 - 17h + 72$ $(h - 8)(h - 9)$

Also available online

Math Humor

Q: What is the hidden math term?

NOMIAL
NOMIAL

A: Binomial.

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

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COMMON CORE

7-4 Factoring $ax^2 + bx + c$

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objective
Factor quadratic trinomials of the form $ax^2 + bx + c$.

Why learn this?
The height of a football that has been kicked can be modeled by a factored polynomial. (See Exercise 69.)

You have already factored trinomials of the form $x^2 + bx + c$. Now you will factor trinomials of the form $ax^2 + bx + c$, where $a \neq 0$ or 1.

When you multiply $(3x + 2)(2x + 5)$, the coefficient of the x^2-term is the product of the coefficients of the x-terms. Also, the constant term in the trinomial is the product of the constants in the binomials.

$$(3x+2)(2x+5) = 6x^2 + 19x + 10$$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, write two sets of parentheses: $(\square x + \square)(\square x + \square)$.

Write two numbers that are factors of a next to the x's and two numbers that are factors of c in the other blanks. Then multiply to see if the product is the original trinomial. If there are not two such integers, the trinomial is unfactorable.

EXAMPLE 1 Factoring $ax^2 + bx + c$ by Guess and Check

Factor $4x^2 + 16x + 15$ by guess and check.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(\square x + \square)(\square x + \square)$ *The first term is $4x^2$, so at least one variable term has a coefficient other than 1.*

The coefficient of the x^2-term is 4. The constant term in the trinomial is 15.

$(1x + 15)(4x + 1) = 4x^2 + 61x + 15$ ✗
$(1x + 5)(4x + 3) = 4x^2 + 23x + 15$ ✗
$(1x + 3)(4x + 5) = 4x^2 + 17x + 15$ ✗
$(1x + 1)(4x + 15) = 4x^2 + 19x + 15$ ✗
$(2x + 15)(2x + 1) = 4x^2 + 32x + 15$ ✗
$(2x + 5)(2x + 3) = 4x^2 + 16x + 15$ ✓

Try factors of 4 for the coefficients and factors of 15 for the constant terms.

The factors of $4x^2 + 16x + 15$ are $(2x + 5)$ and $(2x + 3)$.

$4x^2 + 16x + 15 = (2x + 5)(2x + 3)$

1a. $(3x + 1)(2x + 3)$
1b. $(3x + 4)(x - 2)$

CHECK IT OUT! **Factor each trinomial by guess and check.**

1a. $6x^2 + 11x + 3$ **1b.** $3x^2 - 2x - 8$

AP Photo/Peter Cosgrove

1 Introduce

EXPLORATION

Factoring $ax^2 + bx + c$

You will need algebra tiles for this Exploration.

You can use algebra tiles to factor trinomials in the form $ax^2 + bx + c$.

For example, to factor $2x^2 + 5x + 3$, first use tiles to model the trinomial.

Then arrange the tiles to form a rectangle as shown.

The dimensions of the rectangle give the factorization: $2x^2 + 5x + 3 = (x + 1)(2x + 3)$.

Use algebra tiles to factor each of the following trinomials.

1. $2x^2 + 7x + 3$ $(2x + 1)(x + 3)$
2. $2x^2 + 9x + 10$ $(2x + 5)(x + 2)$
3. $4x^2 + 4x + 1$ $(2x + 1)(2x + 1)$

THINK AND DISCUSS

4. **Explain** how the coefficients of the x-terms in the binomial factors are related to the coefficient of the x^2-term in the trinomial.
5. **Describe** one way in which factoring a trinomial in the form $ax^2 + bx + c$, where $a \neq 0$, is similar to factoring a trinomial in the form $x^2 + bx + c$.

4. The product of the coefficients of the x-terms in the binomial factors is the

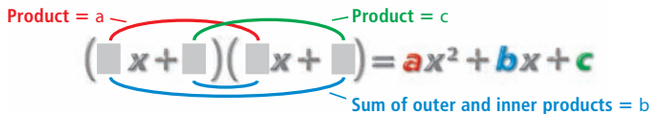
Motivate

Write $6x^2 + 7x + 2$ on the board. Ask students how it is different from the trinomials they have previously factored. The x^2-term has a coefficient that is not 1.

Tell students that because b depends on both a and c, this means that more factors, and their placements, will need to be tested.

Explorations and answers are provided in your online resources.

So, to factor $ax^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b.



Since you need to check all the factors of a and all the factors of c, it may be helpful to make a table. Then check the products of the outer and inner terms to see if the sum is b. You can multiply the binomials to check your answer.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $2x^2 + 11x + 12$

$(\square x + \square)(\square x + \square)$ *a = 2 and c = 12; Outer + Inner = 11*

Factors of 2	Factors of 12	Outer + Inner	
1 and 2	1 and 12	$1(12) + 2(1) = 14$	✗
1 and 2	12 and 1	$1(1) + 2(12) = 25$	✗
1 and 2	2 and 6	$1(6) + 2(2) = 10$	✗
1 and 2	6 and 2	$1(2) + 2(6) = 14$	✗
1 and 2	3 and 4	$1(4) + 2(3) = 10$	✗
1 and 2	4 and 3	$1(3) + 2(4) = 11$	✓

$(x + 4)(2x + 3)$

Check $(x + 4)(2x + 3) = 2x^2 + 3x + 8x + 12$ *Use the FOIL method.*
$= 2x^2 + 11x + 12$ ✓

B $5x^2 - 14x + 8$

$(\square x + \square)(\square x + \square)$ *a = 5 and c = 8; Outer + Inner = −14*

Factors of 5	Factors of 8	Outer + Inner	
1 and 5	−1 and −8	$1(-8) + 5(-1) = -13$	✗
1 and 5	−8 and −1	$1(-1) + 5(-8) = -41$	✗
1 and 5	−2 and −4	$1(-4) + 5(-2) = -14$	✓

$(x - 2)(5x - 4)$

Check $(x - 2)(5x - 4) = 5x^2 - 4x - 10x + 8$ *Use the FOIL method.*
$= 5x^2 - 14x + 8$ ✓

Remember!
When b is negative and c is positive, the factors of c are both negative.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

2a. $6x^2 + 17x + 5$ $(2x + 5)(3x + 1)$

2b. $9x^2 - 15x + 4$ $(3x - 4)(3x - 1)$

2c. $3x^2 + 13x + 12$ $(3x + 4)(x + 3)$

When c is negative, one factor of c will be positive and the other factor will be negative. Only some of the factors are shown in the examples, but you may need to check all of the possibilities.

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Additional Examples

Example 1

Factor $6x^2 + 11x + 4$.
$(3x + 4)(2x + 1)$

Example 2

Factor each trinomial. Check your answer.

A. $2x^2 + 17x + 21$
$(2x + 3)(x + 7)$

B. $3x^2 - 16x + 16$
$(x - 4)(3x - 4)$

Also available online

Mathematical Practices Questioning Strategies

EXAMPLE 1

- How is factoring a trinomial in the form $ax^2 + bx + c$ similar to factoring a trinomial in the form $x^2 + bx + c$? How is it different?

EXAMPLE 2

- What is true about the signs of the factors of the last term if the middle term is positive? negative?
- What should the sum of the inner and outer products be?

Teaching Tip **Inclusion** Previously, confident students may have been able to factor trinomials in their heads. Remind students that careless errors are easy to make. To factor the trinomials in this lesson, these students may need to write out their work.

2 Teach

Guided Instruction

Review factoring trinomials of the form $x^2 + bx + c$. Then have students factor trinomials of the form $ax^2 + bx + c$ by the guess-and-check method. For **Examples 2, 3,** and **4,** use a table when teaching, and encourage students to use a table when working on their own. This will allow students to see what factors they have already checked and prevent them from trying the same sets of factors more than once.

Reaching All Learners *Through Critical Thinking*

Present students with the following trinomials. Challenge students to find values of k for each trinomial that will make the trinomial factorable.

1. $x^2 + kx + 81$ $\pm 82, \pm 30, \pm 18$

2. $x^2 + 14x + k$ $13, 24, 33, \ldots, n(14 - n)$

3. $kx^2 + 10x + 1$ $9, 16, 21, 24, 25$

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Additional Examples

Example 3

Factor each trinomial. Check your answer.

A. $3n^2 + 11n - 4$
$(n + 4)(3n - 1)$

B. $2x^2 + 9x - 18$
$(x + 6)(2x - 3)$

C. $4x^2 - 15x - 4$
$(x - 4)(4x + 1)$

Also available online

Questioning Strategies

EXAMPLE 3

- What do you know about the binomials in the factored expression when the constant term in the trinomial is negative?

Teaching Tip **Multiple Representations** Show students that any trinomial can be written as a polynomial with four terms. It can then be factored by grouping. To factor $6x^2 + 19x + 15$, find ac and factors of ac that sum to b. Rewrite the trinomial using those factors: $6x^2 + 10x + 9x + 15$. Then factor by grouping.
$(6x^2 + 10x) + (9x + 15)$
$= 2x(3x + 5) + 3(3x + 5)$
$= (3x + 5)(2x + 3)$

EXAMPLE 3 Factoring $ax^2 + bx + c$ When c Is Negative

Factor each trinomial. Check your answer.

A $4y^2 + 7y - 2$

$(\square y + \square)(\square y + \square)$ *a = 4 and c = −2; Outer + Inner = 7*

Factors of 4	Factors of −2	Outer + Inner	
1 and 4	1 and −2	$1(-2) + 4(1) = 2$	✗
1 and 4	−1 and 2	$1(2) + 4(-1) = -2$	✗
1 and 4	2 and −1	$1(-1) + 4(2) = 7$	✓

$(y + 2)(4y - 1)$

Check $(y + 2)(4y - 1) = 4y^2 - y + 8y - 2$ *Use the FOIL method.*
$= 4y^2 + 7y - 2$ ✓

B $4x^2 + 19x - 5$

$(\square x + \square)(\square x + \square)$ *a = 4 and c = −5; Outer + Inner = 19*

Factors of 4	Factors of −5	Outer + Inner	
1 and 4	1 and −5	$1(-5) + 4(1) = -1$	✗
1 and 4	−1 and 5	$1(5) + 4(-1) = 1$	✗
1 and 4	5 and −1	$1(-1) + 4(5) = 19$	✓

$(x + 5)(4x - 1)$

Check $(x + 5)(4x - 1) = 4x^2 - x + 20x - 5$ *Use the FOIL method.*
$= 4x^2 + 19x - 5$ ✓

C $2x^2 - 7x - 15$

$(\square x + \square)(\square x + \square)$ *a = 2 and c = −15; Outer + Inner = −7*

Factors of 2	Factors of −15	Outer + Inner	
1 and 2	1 and −15	$1(-15) + 2(1) = -13$	✗
1 and 2	−1 and 15	$1(15) + 2(-1) = 13$	✗
1 and 2	3 and −5	$1(-5) + 2(3) = 1$	✗
1 and 2	−3 and 5	$1(5) + 2(-3) = -1$	✗
1 and 2	5 and −3	$1(-3) + 2(5) = 7$	✗
1 and 2	−5 and 3	$1(3) + 2(-5) = -7$	✓

$(x - 5)(2x + 3)$

Check $(x - 5)(2x + 3) = 2x^2 + 3x - 10x - 15$ *Use the FOIL method.*
$= 2x^2 - 7x - 15$ ✓

CHECK IT OUT! **Factor each trinomial. Check your answer.**

3a. $6x^2 + 7x - 3$
$(3x - 1)(2x + 3)$

3b. $4n^2 - n - 3$
$(4n + 3)(n - 1)$

Student to Student — Factoring $ax^2 + bx + c$

Reggie Wilson
Franklin High School

When a, b, and c are positive, I like to use a box to help me factor. I look for factors of ac that add to b. Then I arrange the terms in a box and factor.

To factor $6x^2 + 7x + 2$, first I find the factors I need.

$ac = 2(6) = 12 \quad b = 7$

Factors of 12	Sum
1 and 12	13
2 and 6	8
3 and 4	7

Then I rewrite the trinomial as $6x^2 + 3x + 4x + 2$.

Now I arrange $6x^2 + 3x + 4x + 2$ in a box and factor out the common factors from each row and column.

$6x^2$	$3x$	→ $3x$
$4x$	2	→ 2
↓ $2x$	↓ 1	

The factors are $(2x + 1)$ and $(3x + 2)$.

When the leading coefficient is negative, factor out -1 from each term before using other factoring methods.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-2x^2 - 15x - 7$.

$-1(2x^2 + 15x + 7)$ — *Factor out -1.*

$-1(\square x + \square)(\square x + \square)$ — *$a = 2$ and $c = 7$; Outer + Inner = 15*

Factors of 2	Factors of 7	Outer + Inner	
1 and 2	1 and 7	$1(7) + 2(1) = 9$	✗
1 and 2	7 and 1	$1(1) + 2(7) = 15$	✓

$(x + 7)(2x + 1)$

$-1(x + 7)(2x + 1)$

Caution!
When you factor out -1 in an early step, you must carry it through the rest of the steps.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

4a. $-6x^2 - 17x - 12$
$-1(2x + 3)(3x + 4)$

4b. $-3x^2 - 17x - 10$
$-1(3x + 2)(x + 5)$

THINK AND DISCUSS

1. Let a, b, and c be positive. If $ax^2 + bx + c$ is the product of two binomials, what do you know about the signs of the numbers in the binomials?

2. GET ORGANIZED Copy and complete the graphic organizer. Write each of the following trinomials in the appropriate box and factor each one.

$3x^2 + 10x - 8$ $\quad 3x^2 + 10x + 8$
$3x^2 - 10x + 8$ $\quad 3x^2 - 10x - 8$

Factoring $ax^2 + bx + c$	
$c > 0$	
$b > 0$	$b < 0$
$c < 0$	
$b < 0$	$b > 0$

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COMMON ERROR ALERT

Students sometimes forget that every number has 1 and itself as two of its factors. When students are making their lists of factors, remind them to try all possible factors.

Power Presentations with PowerPoint®

Additional Examples

Example 4

Factor $-2x^2 - 5x - 3$.
$-1(x + 1)(2x + 3)$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- How do you know which term is the leading coefficient?
- What is the advantage to factoring out -1 in the first step?

3 Close

Summarize

Tell students that to factor a trinomial in the form $ax^2 + bx + c$, they should make a table, such that each row has two factors of a, two factors of c, and the sum of the inner and outer products. Remind them that if they factor out -1, they need to remember to put it in the factored form. It doesn't just disappear.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. They are all positive.
2. See Additional Answers.

7-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 25–39, 52, 55, 58–64 even
- **Average** 25–39, 58–64 even, 81, 83
- **Advanced** 25–39, 58–64 even, 81, 83

If you finished Examples **1–4**
- **Basic** 25–64, 68–74, 76–80
- **Average** 25–70, 75–81, 83
- **Advanced** 25–51, 52–62 even, 64–69, 75–89

Homework Quick Check
Quickly check key concepts.
Exercises: 30, 35, 48, 50, 58, 64

Answers

1. $(2x + 5)(x + 2)$
2. $(x + 6)(5x + 1)$
3. $(5x - 3)(x + 2)$
4. $(x + 6)(6x + 1)$
5. $(3x + 4)(x - 6)$
6. $(2x - 1)(3x + 2)$
7. $(x + 2)(5x + 1)$
8. $(x + 5)(2x + 1)$
9. $(4x - 5)(x - 1)$
10. $(2y - 7)(y - 2)$
11. $(5x + 4)(x + 1)$
12. $(x + 2)(3x + 1)$

13–63. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 68–69, 71

Reason abstractly and quantitatively. Exercises 87–89

Construct viable arguments and critique the reasoning of others. Exercises 65, 70, 76

Model with mathematics. Exercises 52–54, 64

Look for and make use of structure. Exercises 66–67, 72–75

Learn It Online
State Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Factor each trinomial by guess and check.

1. $2x^2 + 9x + 10$
2. $5x^2 + 31x + 6$
3. $5x^2 + 7x - 6$
4. $6x^2 + 37x + 6$
5. $3x^2 - 14x - 24$
6. $6x^2 + x - 2$

Factor each trinomial. Check your answer.

SEE EXAMPLE 2

7. $5x^2 + 11x + 2$
8. $2x^2 + 11x + 5$
9. $4x^2 - 9x + 5$
10. $2y^2 - 11y + 14$
11. $5x^2 + 9x + 4$
12. $3x^2 + 7x + 2$

SEE EXAMPLE 3

13. $4a^2 + 8a - 5$
14. $15x^2 + 4x - 3$
15. $2x^2 + x - 6$
16. $6n^2 - 11n - 10$
17. $10x^2 - 9x - 1$
18. $7x^2 - 3x - 10$

SEE EXAMPLE 4

19. $-2x^2 + 5x + 12$
20. $-4n^2 - 16n + 9$
21. $-5x^2 + 7x + 6$
22. $-6x^2 + 13x - 2$
23. $-4x^2 - 8x + 5$
24. $-5x^2 + x + 18$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
25–33	1
34–42	2
43–48	3
49–51	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Factor each trinomial by guess and check.

25. $9x^2 + 9x + 2$
26. $2x^2 + 7x + 5$
27. $3n^2 + 8n + 4$
28. $10d^2 + 17d + 7$
29. $4c^2 - 17c + 15$
30. $6x^2 + 14x + 4$
31. $8x^2 + 22x + 5$
32. $6x^2 - 13x + 6$
33. $5x^2 + 9x - 18$

Factor each trinomial. Check your answer.

34. $6x^2 + 23x + 7$
35. $10n^2 - 17n + 7$
36. $3x^2 + 11x + 6$
37. $7x^2 + 15x + 2$
38. $3n^2 + 4n + 1$
39. $3x^2 - 19x + 20$
40. $6x^2 + 11x + 4$
41. $4x^2 - 31x + 21$
42. $10x^2 + 31x + 15$
43. $12y^2 + 17y - 5$
44. $3x^2 + 10x - 8$
45. $4x^2 + 4x - 3$
46. $2n^2 - 7n - 4$
47. $3x^2 - 4x - 15$
48. $3n^2 - n - 4$
49. $-4x^2 - 4x + 15$
50. $-3x^2 + 16x - 16$
51. $-3x^2 - x + 2$

Geometry For Exercises 52–54, write the polynomial modeled and then factor.

52.

$12x^2$	$24x$
$3x$	6

53.

$2x^2$	$-x$
$-4x$	2

54.

$5x^2$	$-4x$
$35x$	-28

Factor each trinomial, if possible.

55. $9n^2 + 17n + 8$
56. $2x^2 - 7x - 4$
57. $4x^2 - 12x + 5$
58. $5x^2 - 4x + 12$
59. $3x^2 + 14x + 16$
60. $-3x^2 - 11x + 4$
61. $6x^2 - x - 12$
62. $10a^2 + 11a + 3$
63. $4x^2 - 12x + 9$

READING STRATEGIES

Most people factor trinomials of the form $ax^2 + bx + c$ using a sort of organized guess-and-check method. Look at the model below.

Factor the trinomial $4x^2 - 17x - 15$ into two binomials.

- **Recall:** The acronym FOIL stands for First, Outer, Inner, Last.
- **Signs:** This trinomial requires a "plus" and a "minus" binomial.
- **Setup:** (F + L) (F – L) → (▢x + ▢) (▢x – ▢)
- **"First":** The factorizations of 4 are: 1 • 4 and 2 • 2.
- **"Last":** The factorizations of 15 are: 1 • 15 and 3 • 5.
- **"Outer" and "Inner":** Guess-and-check these values in the binomials. The goal is to multiply the "Outer" terms and the "Inner" terms, and get a sum of –17.

(x + ▢)(x – ▢)

				Outer	+	Inner			
1	1	4	–15	1 • –15	+	1 • 4	=	–11	No
1	15	4	–1	1 • –1	+	15 • 4	=	59	No
1	5	4	–3	1 • –3	+	5 • 4	=	–7	No
1	3	4	–5	1 • –5	+	3 • 4	=	7	No
4	1	1	–15	4 • –15	+	1 • 1	=	–59	No
4	15	1	–1	4 • –1	+	15 • 1	=	11	No
4	5	1	–3	4 • –3	+	5 • 1	=	–7	No
4	**3**	**1**	**–5**	4 • –5	+	3 • 1	=	–17	Yes

- **Answer:** $4x^2 - 17x - 15$ factors into $(4x + 3)(1x - 5)$, or just $(4x + 3)(x - 5)$.

Complete the following to guide you in factoring $3x^2 - 20x + 12$.

1. What are the factorizations of 3? 1 ▢ 3
2. What are the factorizations of 12? 1 ▢ 12; 2 ▢ 6; 3 ▢ 4
3. What signs are needed in each of the two binomials? minus, minus
4. Set up the binomials for factoring this polynomial. (x –)(x –)
5. Guess-and-check the factors that will result in the correct middle term coefficient of 20.

Possible answer:

(↓ x – ↓)(↓ x – ↓)

				Outer	+	Inner			
3	–1	1	–12	3 ▢ –12	+	–1 ▢ 1	=	–37	No
3	–12	1	–1	3 ▢ –1	+	–12 ▢ 1	=	–15	No
3	–4	1	–3	3 ▢ –3	+	–4 ▢ 1	=	–13	No
3	–3	1	–4	3 ▢ –4	+	–3 ▢ 1	=	–15	No
3	–6	1	–2	3 ▢ –2	+	–6 ▢ 1	=	–12	No
3	–2	1	–6	3 ▢ –6	+	–2 ▢ 1	=	–20	Yes

6. What is the final answer for factoring $3x^2 - 20x + 12$? $(3x - 2)(x - 6)$

RETEACH

When factoring $ax^2 + bx + c$, first find factors of a and c. Then check the products of the inner and outer terms to see if the sum is b.

Factor $2x^2 + 11x + 15$. Check your answer.

$2x^2 + 11x + 15 = (▢x + ▢)(▢x + ▢)$

Factors of 2	Factors of 15	Outer + Inner
1 and 2	1 and 15	1 • 15 + 2 • 1 = 17 ×
1 and 2	15 and 1	1 • 1 + 2 • 15 = 31 ×
1 and 2	5 and 3	1 • 3 + 2 • 5 = 13 ×
1 and 2	3 and 5	1 • 5 + 2 • 3 = 11 ✓

$(x + 3)(2x + 5)$

Check:
$(x + 3)(2x + 5) = 2x^2 + 5x + 6x + 15$
$= 2x^2 + 11x + 15$ ✓

Factor $3x^2 - 23x + 14$. Check your answer.

$3x^2 - 23x + 14 = (▢x + ▢)(▢x + ▢)$

Factors of 3	Factors of 14	Outer + Inner
1 and 3	–1 and –14	1 • (–14) + 3 • (–1) = –17 ×
1 and 3	–14 and –1	1 • (–1) + 3 • (–14) = –42 ×
1 and 3	–2 and –7	1 • (–7) + 3 • (–2) = –13 ×
1 and 3	–7 and –2	1 • (–2) + 3 • (–7) = –23 ✓

$(x - 7)(3x - 2)$

Check:
$(x - 7)(3x - 2) = 3x^2 - 2x - 21x + 14$
$= 3x^2 + 23x + 14$ ✓

1. Factor $5x^2 + 12x + 4$ by filling in the blanks below.

Factors	Factors	Outer + Inner
1 and 5	1 and 4	1 • 4 + 5 • 1 = 9
1 and 5	4 and 1	1 • 1 + 5 • 4 = 21
1 and 5	2 and 2	1 • 2 + 5 • 2 = 12

$(x + 2)(5x + 2)$

Factor each trinomial.

2. $3x^2 + 7x + 4$ $(3x + 4)(x + 1)$
3. $2x^2 - 13x + 21$ $(2x - 7)(x - 3)$
4. $4x^2 + 8x + 3$ $(2x + 3)(2x + 1)$

64. Geometry The area of a rectangle is $(6x^2 + 11x + 5)$ cm^2. The width is $(x + 1)$ cm. What is the length of the rectangle? **$(6x + 5)$ cm**

$(x + 1)$ cm

65. Write About It Write a paragraph describing how to factor $6x^2 + 13x + 6$. Show each step you would take and explain your steps. **$(2x + 3)(3x + 2)$; Check students' work.**

Complete each factorization.

66. $8x^2 + 18x - 5$

$8x^2 + 20x - 2x - 5$

$(8x^2 + 20x) - (2x + 5)$

$4x$ $(2x$ $+ 5)$ $- 1$ $(2x + 5)$

$(4x - 1)(2x + 5)$

67. $4x^2 + 9x + 2$

$4x^2 + 8x + x + 2$

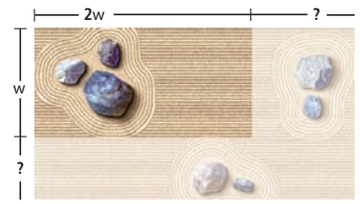
$(4x^2 + 8x) + (x + 2)$

$4x$ $(x$ $+ 2)$ $+ 1$ $(x + 1)$

$(4x + 1)(x + 2)$

68. width increased by 2 yd; length increased by 3 yd

68. Gardening The length of Rebecca's rectangular garden was two times the width w. Rebecca increased the length and width of the garden so that the area of the new garden is $(2w^2 + 7w + 6)$ square yards. By how much did Rebecca increase the length and the width?



69. Physics The height of a football that has been thrown or kicked can be described by the expression $-16t^2 + vt + h$ where t is the time in seconds, v is the initial upward velocity, and h is the initial height in feet.

a. Write an expression for the height of a football at time t when the initial upward velocity is 20 feet per second and the initial height is 6 feet. **$-16t^2 + 20t + 6$**

b. Factor your expression from part **a**. **$-2(4t + 1)(2t - 3)$**

c. Find the height of the football after 1 second. **10 ft**

70. ERROR ANALYSIS A student attempted to factor $2x^2 + 11x + 12$ as shown. Find and explain the error.

Possible answer: The student tried factors of 12, but forgot to try factors of 2 also.

$2x^2 + 11x + 12$

Factors of 12	Sum	
1 and 12	13	✓
2 and 6	8	x
3 and 4	7	x

$(2x + 1)(x + 12)$

MULTI-STEP TEST PREP

71. The equation $d = 2t^2$ gives the distance from the start point of a toy boat that starts at rest and accelerates at 4 cm/s^2. The equation $d = 10t - 8$ gives the distance from the start point of a second boat that starts at rest 8 cm behind the first boat and travels at a constant rate of 10 cm/s.

a. $2t^2 - 10t + 8 = 0$

a. By setting the equations equal to each other, you can determine when the cars are the same distance from the start point: $2t^2 = 10t - 8$. Use properties of algebra to collect all terms on the left side of the equation, leaving 0 on the right side.

b. Factor the expression on the left side of the equation. **$2(t - 1)(t - 4)$**

c. The boats are the same distance from the start point at $t = 1$ and $t = 4$. Explain how the factors you found in part **b** were used to find these two times.
$2(t - 4)(t - 1) = 0$ so $t - 4 = 0$ or $t - 1 = 0$

HMH

Teaching Tip

Geometry Link In **Exercise 68,** have students draw a rectangle and label it with the original dimensions of the garden: w and $2w$. This will make it easier for them to see which factor of the factored expression is the length and which is the width.

MULTI-STEP TEST PREP

Exercise 71 involves solving a quadratic equation by factoring. This exercise prepares students for the Multi-Step Test Prep.

PRACTICE A

PRACTICE C

PRACTICE B

Factor each trinomial.

1. $2x^2 + 13x + 15$ — **$(2x + 3)(x + 5)$**
2. $3x^2 + 10x + 8$ — **$(3x + 4)(x + 2)$**
3. $4x^2 + 24x + 27$ — **$(2x + 9)(2x + 3)$**
4. $5x^2 + 21x + 4$ — **$(5x + 1)(x + 4)$**
5. $4x^2 + 11x + 7$ — **$(4x + 7)(x + 1)$**
6. $6x^2 - 23x + 20$ — **$(3x - 4)(2x - 5)$**
7. $7x^2 - 59x + 24$ — **$(7x - 3)(x - 8)$**
8. $3x^2 - 14x + 15$ — **$(3x - 5)(x - 3)$**
9. $8x^2 - 73x + 9$ — **$(8x - 1)(x - 9)$**
10. $2x^2 + 11x - 13$ — **$(2x + 13)(x - 1)$**
11. $3x^2 + 2x - 16$ — **$(3x + 8)(x - 2)$**
12. $2x^2 + 17x - 30$ — **$(x + 10)(2x - 3)$**
13. $8x^2 + 29x - 12$ — **$(x + 4)(8x - 3)$**
14. $11x^2 + 25x - 24$ — **$(x + 3)(11x - 8)$**
15. $9x^2 - 3x - 2$ — **$(3x + 1)(3x - 2)$**
16. $12x^2 - 7x - 12$ — **$(4x + 3)(3x - 4)$**
17. $9x^2 - 49x - 30$ — **$(9x + 5)(x - 6)$**
18. $6x^2 + x - 40$ — **$(3x + 8)(2x - 5)$**
19. $-12x^2 - 35x - 18$ — **$-1(4x + 9)(3x + 2)$**
20. $-20x^2 + 29x - 6$ — **$-1(5x - 6)(4x - 1)$**
21. $-2x^2 + 5x + 42$ — **$-1(2x + 7)(x - 6)$**
22. The area of a rectangle is $20x^2 - 27x - 8$. The length is $4x + 1$. What is the width? **$5x - 8$**

PROBLEM SOLVING

Write the correct answer.

1. A rectangular painting has an area of $(2x^2 + 8x + 6)$ cm^2. Its length is $(2x + 2)$ cm. Find the width of the painting. **$(x + 3)$ cm**
2. A ball is kicked straight up into the air. The height of the ball in feet is given by the expression $-16t^2 + 12t + 4$, where t is time in seconds. Factor the expression. Then find the height of the ball after 1 second. **$-1(4t - 4)(4t + 1)$ or $-4(4t + 1)(t - 1)$; 0 feet**
3. Instructors led an exercise class from a raised rectangular platform at the front of the room. The width of the platform was $(3x - 1)$ feet and the area was $(9x^2 + 6x - 3)$ ft^2. Find the length of this platform. After the exercise studio is remodeled, the area of the platform will be $(9x^2 + 12x + 3)$ ft^2. By how many feet will the width of the platform change? **$(3x + 3)$ ft; increase of 2 ft**
4. A clothing store has a rectangular clearance section with a length that is twice the width w. During a sale, the section is expanded to an area of $(2w^2 + 19w + 35)$ ft^2. Find the amount of the increase in the length and width of the clearance section. **length increased by 5 ft, width increased by 7 ft**

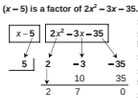
Select the best answer.

5. The area of a soccer field is $(24x^2 + 100x + 100)$ m^2. The width of the field is $(4x + 10)$ m. What is the length?
   A $(3x + 10)$ m C $(6x + 10)$ m
   B $(6x + 1)$ m D $(8x + 2)$ m
6. A square parking lot has an area of $(4x^2 + 20x + 25)$ ft^2. What is the length of one side of the parking lot?
   F $(2x + 5)$ ft H $(5x + 4)$ ft
   G $(2x + 10)$ ft J $(5x + 2)$ ft
7. For a certain college, the number of applications received after x recruiting seminars is modeled by the polynomial $3x^2 + 490x + 6000$. What is this expression in factored form?
   A $(3x - 40)(x - 150)$
   B $(3x + 40)(x + 150)$
   C $(3x - 30)(x - 200)$
   D $(3x - 30)(x + 200)$
8. Jin needs to fence in his rectangular backyard. The fence will have one long section away from, but parallel to, the length of his house and two shorter sides connecting that section to the house. The length of Jin's house is $(3x + 4)$ yd and the area of his backyard is $(9x^2 + 15x + 4)$ yd^2. How many yards of fencing will Jin need?
   F $(6x + 2)$ yd H $(9x + 9)$ yd
   G $(9x + 6)$ yd J $(12x + 10)$ yd

CHALLENGE

Synthetic division is a quick way to divide a polynomial by a binomial in the form $x - a$ yielding another factor of the polynomial.

$(x - 5)$ is a factor of $2x^2 - 3x - 35$. Use synthetic division to find the other factor.



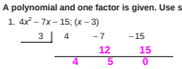
Step 1: Write the coefficients of the polynomial (in standard form). Write a in a box to the left. Draw a line below the coefficients.
Step 2: Write the first coefficient below the line.
Step 3: Multiply that coefficient by the value of a and write the product under the next coefficient.
Step 4: Add the numbers in the second column.
Step 5: Repeat until all additions are completed.

The other factor is $2x + 7$.

If done correctly, the last sum will be zero. The numbers under the bar are the coefficients of the quotient, in decreasing degree.

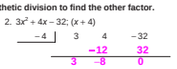
A polynomial and one factor is given. Use synthetic division to find the other factor.

1. $4x^2 - 7x - 15$; $(x - 3)$



$(5x + 5)$

2. $3x^2 + 4x - 32$; $(x + 4)$

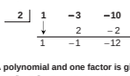


$(3x - 8)$

3. $6x^2 - 29x - 5$; $(x - 5)$ — **$(6x + 1)$**
4. $8x^2 + 11x - 10$; $(x + 2)$ — **$(8x - 5)$**

Some polynomials have three factors.

$(x - 2)$ is a factor of $x^3 - 3x^2 - 10x + 24$. Factor completely.



Now factor $x^2 - x - 12$ using a method you know.
$x^2 - x - 12 = (x + 3)(x - 4)$
The factors are: $(x - 2)(x + 3)(x - 4)$.

A polynomial and one factor is given. Factor completely.

5. $x^3 + 7x^2 + 7x - 15$; $(x - 1)$ — **$(x - 1)(x + 3)(x + 5)$**
6. $2x^3 - 3x^2 - 39x + 20$; $(x + 4)$ — **$(x + 4)(2x - 1)(x - 5)$**

TEST PREP DOCTOR In **Exercise 78,** 1 and 5 are the only choices for the factors of a. Choices **F** and **J** can be eliminated.

Tell students that they can immediately eliminate **C** and **D** in **Exercise 79.** Because b is negative and c is positive, the constants will be subtracted in both binomials.

Answers

81. $(2x + 1)(2x + 1)$
82. $(7x - 1)(7x - 1)$
83. $(9x + 1)(9x + 1)$
84. $(3x + 5)(3x + 5)$
85. $(5x + 2)(5x + 2)$
86. $(3x - 2)(3x - 2)$

Match each trinomial with its correct factorization.

72. $6x^2 - 29x - 5$ **D** A. $(x + 5)(6x + 1)$
73. $6x^2 - 31x + 5$ **B** B. $(x - 5)(6x - 1)$
74. $6x^2 + 31x + 5$ **A** C. $(x + 5)(6x - 1)$
75. $6x^2 + 29x - 5$ **C** D. $(x - 5)(6x + 1)$

76. **Critical Thinking** The quadratic trinomial $ax^2 + bx + c$ has $a > 0$ and can be factored into the product of two binomials.
   a. Explain what you know about the signs of the constants in the factors if $c > 0$.
   b. Explain what you know about the signs of the constants in the factors if $c < 0$.

76a. Both signs are pos., or both signs are neg.

b. One sign is pos., and the other is neg.

TEST PREP

77. What value of b would make $3x^2 + bx - 8$ factorable?
   Ⓐ 3 Ⓑ 10 Ⓒ 11 Ⓓ 25

78. Which product of binomials is represented by the model?

$5x^2$	$4x$
$15x$	12

   Ⓕ $(x + 4)(3x + 5)$ Ⓗ $(x + 3)(5x + 4)$
   Ⓖ $(x + 4)(5x + 3)$ Ⓙ $(x + 5)(3x + 4)$

79. Which binomial is a factor of $24x^2 - 49x + 2$?
   Ⓐ $x - 2$ Ⓑ $x - 1$ Ⓒ $x + 1$ Ⓓ $x + 2$

80. Which value of c would make $2x^2 + x + c$ NOT factorable?
   Ⓕ -15 Ⓖ -9 Ⓗ -6 Ⓙ -1

CHALLENGE AND EXTEND

Factor each trinomial. Check your answer.

81. $1 + 4x + 4x^2$
82. $1 - 14x + 49x^2$
83. $1 + 18x + 81x^2$
84. $25 + 30x + 9x^2$
85. $4 + 20x + 25x^2$
86. $4 - 12x + 9x^2$
87. Find all possible values of b such that $3x^2 + bx + 2$ can be factored. **$-7; -5; 5; 7$**
88. Find all possible values of b such that $3x^2 + bx - 2$ can be factored. **$-5; -1; 1; 5$**
89. Find all possible values of b such that $5x^2 + bx + 1$ can be factored. **$-6; 6$**

Journal

Have students explain how to factor a trinomial of the form $ax^2 + bx + c$ when a is negative.

ALTERNATIVE ASSESSMENT

Have students write binomials of the form $(x + a)$, $(x - a)$, $(ax + b)$, and $(ax - b)$ and then make six trinomials by multiplying each different pair. Have students give their trinomials to another student to factor.

Power Presentations with PowerPoint®

7-4 Lesson Quiz

Factor each trinomial. Check your answer.

1. $5x^2 + 17x + 6$
   $(5x + 2)(x + 3)$
2. $2x^2 + 5x - 12$
   $(2x - 3)(x + 4)$
3. $6x^2 - 23x + 7$
   $(3x - 1)(2x - 7)$
4. $-4x^2 + 11x + 20$
   $(-x + 4)(4x + 5)$
5. $-2x^2 + 7x - 3$
   $(-2x + 1)(x - 3)$
6. $8x^2 + 27x + 9$
   $(8x + 3)(x + 3)$

Also available online

7-4 Technology LAB

Use a Graph to Factor Polynomials

Use with Factoring $ax^2 + bx + c$

You can use a graphing calculator to help factor polynomials.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines. *Also* **CC.9-12.A.SSE.2**

Learn It Online
Lab Resources Online

Activity

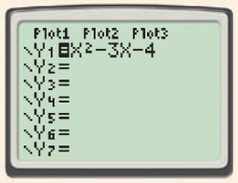
Factor $x^2 - 3x - 4$ using algebra and check your factorization using a graphing calculator.

1. $x^2 - 3x - 4$

   $(x + \square)(x + \square)$ *b = −3 and c = −4; look for factors of −4 whose sum is −3.*

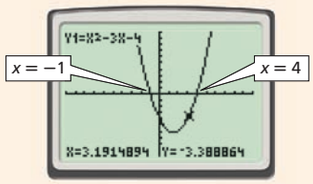
   $(x - 4)(x + 1)$ *−4(1) = −4; −4 + 1 = −3*

2. Press **Y=** and enter $x^2 - 3x - 4$ for **Y1.**



3. Press **GRAPH** to view the graph of the equation.

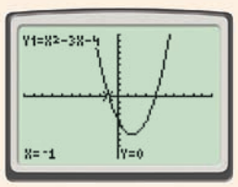
4. Press **TRACE** and use the left and right buttons to move the cursor along the graph. The graph appears to cross the x-axis at $x = -1$ and $x = 4$.



5. To find the value of y at $x = -1$, enter -1 and press **ENTER** while in *Trace* mode. The calculator gives you a value for y. Then enter 4 to find the value of y at $x = 4$.

   The calculator tells you that $y = 0$ at $x = -1$ and at $x = 4$.

   Notice that for a function with a binomial factor of the form $(x - a)$, it appears that a is an x-intercept.



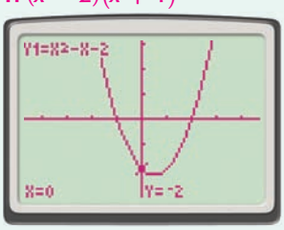
Try This

Graph each trinomial and use the graph to predict the factors. Then factor each trinomial using algebra.

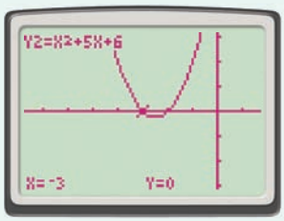
1. $x^2 - x - 2$
2. $x^2 + 5x + 6$
3. $x^2 + x - 12$
4. $x^2 + 12x - 64$
5. $x^2 - 4x - 5$
6. $3x^2 + 16x - 12$

Answers to *Try This*

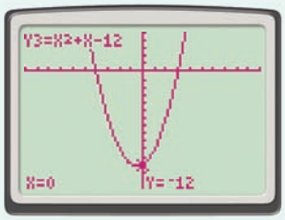
1. $(x - 2)(x + 1)$



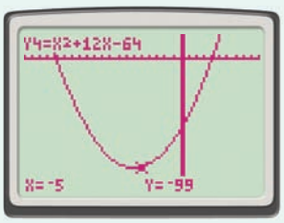
2. $(x + 3)(x + 2)$



3. $(x + 4)(x - 3)$



4. $(x + 16)(x - 4)$



5–6. See Additional Answers.

Technology LAB Organizer

Use with the lesson Factoring $ax^2 + bx + c$

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to factor a polynomial by studying its graph.

Materials: graphing calculator

INTERACTIVE **Online Edition**
Graphing Calculator, TechKeys

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

The factors of c are the opposites of the x-intercepts. Students can graph both the factored and original form of a polynomial and determine whether they have the same graph.

Alternative Approach

Students can use the TABLE feature to find the x-intercepts.

Close

Key Concept

If a is an x-intercept, then $(x - a)$ is a factor of the graphed polynomial.

Assessment

Journal Have students explain how to factor a polynomial by using a graphing calculator.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
a. Factor a quadratic expression to reveal the zeros of the function it defines.
CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Learn It Online
State Resources Online

SECTION 7A

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

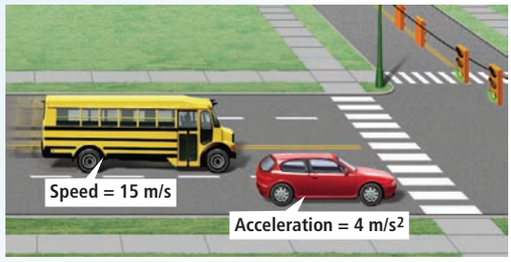
Learn It Online
State Resources Online

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Factoring

Red Light, Green Light The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in meters, v is starting velocity in m/s, a is acceleration in m/s^2, and t is time in seconds.



1. A car is stopped at a traffic light. The light changes to green and the driver starts to drive, accelerating at a rate of 4 m/s^2. Write an equation for the distance the car travels in time t. $d = 2t^2$
2. A bus is traveling at a speed of 15 m/s. The driver approaches the same traffic light in another traffic lane. He does not brake, and continues at the same speed. Write an equation for the distance the bus travels in time t. (*Hint:* At a constant speed, the acceleration is 0 m/s^2.) $d = 15t$
3. Set the equations equal to each other so you can determine when the car and bus are the same distance from the intersection. Collect all the terms on the left side of this new equation, leaving 0 on the right side. Factor the expression on the left side of the equation. $2t^2 - 15t = 0$; $t(2t - 15)$
4. Let $t = 0$ be the point at which the car is just starting to drive and the bus is even with the car. Find the other time when the vehicles will be the same distance from the intersection. $t = 7.5$ s
5. What distance will the two vehicles have traveled when they are again at the same distance from the intersection? $d = 112.5$ m
6. A truck traveling at 16 m/s is 24 meters behind the bus at $t = 0$. The equation $d = -24 + 16t$ gives the position of the truck. At what time will the truck be the same distance from the intersection as the bus? What will that distance be? $t = 24$ s; $d = 360$ m

(br), © Peter Adams/Getty Images; (t), ©Mi

INTERVENTION

Scaffolding Questions

1. What is the initial velocity of an object that is stopped? zero m/s
2. What is the acceleration of an object that does not change velocity? zero m/s^2
3. If the product of two numbers is zero, why is at least one of the numbers zero? Zero multiplied by any number is zero.
4. Explain why one solution to the equation for the car and bus is $t = 0$. The car and bus both start at the intersection with the traffic light.
5. How can you determine the distance traveled by the car? Substitute the time from Problem 4 in the equation from Problem 1.
6. Explain why $t = 0$ is not a solution for the car and truck. The car and truck are not at the same position when the light changes.

Extension

During what period of time is the truck ahead of the car? between $t = 2$ s and $t = 6$ s

READY TO GO ON?

CHAPTER 7

SECTION 7A

Quiz for Lessons 7-1 Through 7-4

7-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 54 $2 \cdot 3^3$
2. 42 $2 \cdot 3 \cdot 7$
3. 50 $2 \cdot 5^2$
4. 120 $2^3 \cdot 3 \cdot 5$
5. 44 $2^2 \cdot 11$
6. 78 $2 \cdot 3 \cdot 13$

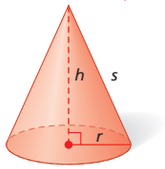
Find the GCF of each pair of monomials.

7. $6p^3$ and $2p$ $2p$
8. $12x^3$ and $18x^4$ $6x^3$
9. -15 and $20s^4$ 5
10. $3a$ and $4b^2$ 1
11. Brent is making a wooden display case for his baseball collection. He has 24 balls from American League games and 30 balls from National League games. He wants to display the same number of baseballs in each row and does not want to put American League baseballs in the same row as National League baseballs. How many rows will Brent need in the display case to put the greatest number of baseballs possible in each row? 9

7-2 Factoring by GCF

Factor each polynomial. Check your answer.

12. $2d^3 + 4d$ $2d(d^2 + 2)$
13. $m^2 - 8m^5$ $m^2(1 - 8m^3)$
14. $12x^4 - 8x^3 - 4x^2$ $4x^2(3x^2 - 2x - 1)$
15. $3k^2 + 6k - 3$ $3(k^2 + 2k - 1)$
16. The surface area of a cone can be found using the expression $s\pi r + \pi r^2$, where s represents the slant height and r represents the radius of the base. Factor this expression. $\pi r(s + r)$



Factor each polynomial by grouping. Check your answer.

17. $w^3 - 4w^2 + w - 4$ $(w^2 + 1)(w - 4)$
18. $3x^3 + 6x^2 - 4x - 8$ $(3x^2 - 4)(x + 2)$
19. $2p^3 - 6p^2 + 15 - 5p$ $(2p^2 - 5)(p - 3)$
20. $n^3 - 6n^2 + 5n - 30$ $(n^2 + 5)(n - 6)$

7-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

21. $n^2 + 9n + 20$
22. $d^2 - 6d - 7$
23. $x^2 - 6x + 8$
24. $y^2 + 7y - 30$
25. $k^2 - 6k + 5$
26. $c^2 - 10c + 24$
27. Simplify and factor the expression $n(n + 3) - 4$. Show that the original expression and the factored form have the same value for $n = 0, 1, 2, 3$, and 4.

7-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

28. $2x^2 + 11x + 5$
29. $3n^2 + 16n + 21$
30. $5y^2 - 7y - 6$
31. $4g^2 - 10g + 6$
32. $6p^2 - 18p - 24$
33. $12d^2 + 7d - 12$
34. The area of a rectangle is $(8x^2 + 8x + 2)$ cm^2. The width is $(2x + 1)$ cm. What is the length of the rectangle? $(4x + 2)$ cm

READY TO GO ON?

SECTION 7A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

21–33. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 7A: Worksheets	*Ready to Go On? Intervention*, Section 7A: Online	
Lesson 7-1	7-1 Intervention	Activity 7-1	Diagnose and Prescribe Online
Lesson 7-2	7-2 Intervention	Activity 7-2	
Lesson 7-3	7-3 Intervention	Activity 7-3	
Lesson 7-4	7-4 Intervention	Activity 7-4	

***Ready to Go On? Enrichment*, Section 7A**
Worksheets
Online

SECTION
7B Applying Factoring Methods

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 7-5 Factoring Special Products • Factor perfect-square trinomials. • Factor the difference of two squares ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** algebra tiles (MK)
Lesson 7-6 Choosing a Factoring Method • Choose an appropriate method for factoring a polynomial. • Combine methods for factoring a polynomial. ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 39, 41a, 41b, 41c, 43a, 43b, 43c, 43d, 43e, 45, 47, 49, 51a, 51b, 53

Lesson 6: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 45, 47, 53, 55, 57, 59a, 59b, 59c, 61, 63

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

FACTORING POLYNOMIALS

Lessons 7-5 and 7-6

Algebra is an extension of arithmetic. As such, there are many parallels between the two. For example, the arithmetic idea of factoring a number can be extended into the realm of algebra by considering factorizations of polynomials.

Factoring polynomials is an essential algebraic skill. As students will see, it provides a powerful method for solving quadratic equations. However, students must first develop an arsenal of techniques for factoring.

FACTORING METHODS

Lesson 7-6

The simplest factoring methods are direct applications of the Distributive Property. In particular, the Distributive Property makes it possible to factor out the greatest common factor (GCF) of the terms of a polynomial.

For example, to factor the polynomial $3x^2 + 12x^3$, we may rewrite it as $3 \cdot x \cdot x + 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x$, where the integer coefficients have been written in their prime factorizations. This form of the polynomial makes it easy to see that the GCF of the terms is $3x^2$, and by the Distributive Property, $3x^2 + 12x^3 = 3x^2(1 + 4x)$.

The Distributive Property is also at work when students use more general methods to factor binomials. To factor the polynomial $x^2 + 7x + 12$, students will look for two factors of 12 whose sum is 7. This leads to the factorization $(x + 4)(x + 3)$. However, it is instructive to look at the intermediate steps. Once the correct factors of 12 have been identified, we can write

1. $x^2 + 7x + 12 = x^2 + 3x + 4x + 12$
2. $= x(x + 3) + 4(x + 3)$
3. $= (x + 4)(x + 3)$

The Distributive Property has been used twice to go from Step 1 to Step 2 and has been applied again to go from Step 2 to Step 3. It is also useful to recognize that the polynomial on the right side of the equation in Step 1 consists of precisely the set of terms that result from multiplying $(x + 4)(x + 3)$ by using the FOIL method. In other words, multiplying polynomials and factoring polynomials are inverse processes based on the Distributive Property.

UNFACTORABLE POLYNOMIALS

Lesson 7-6

Students should understand that some polynomials cannot be factored. For example, the trinomials $x^2 + x + 1$ and $2x^2 + 5x + 4$ are unfactorable. We call these polynomials *prime polynomials*.

Note that when we say a polynomial is unfactorable in an Algebra 1 course, this generally means that it cannot be factored *over the integers*. In other words, the polynomial cannot be written as a product of polynomials of lesser degree whose terms have integer coefficients. However, it may be possible to factor the polynomial over other sets of numbers.

For example, consider the binomial $x^2 - 2$. This binomial is unfactorable over the integers, but it can be factored over the real numbers as $(x + \sqrt{2})(x - \sqrt{2})$. Furthermore, the binomial $x^2 + 4$ cannot be factored over the integers or over the real numbers, but it can be factored over the complex numbers as $(x + 2i)(x - 2i)$, where $i = \sqrt{-1}$. (Students will be introduced to the set of complex numbers in Algebra 2.)

To take this a step further, the Fundamental Theorem of Algebra asserts that every polynomial of degree $n \geq 1$ has n roots in the complex numbers. As a result, every polynomial can be completely factored (that is, written as a product of linear factors) in a unique way over the complex numbers, with the exception of different orderings of the factors. Thus, the Fundamental Theorem of Algebra is analogous to the Fundamental Theorem of Arithmetic, demonstrating once again that algebra is an extension of arithmetic.

7-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Factor perfect-square trinomials.

Factor the difference of two squares.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Determine whether the following are perfect squares. If so, find the positive square root.

1. 64 yes; 8
2. 36 yes; 6
3. 45 no
4. x^2 yes; x
5. y^8 yes; y^4
6. $4x^6$ yes; $2x^3$
7. $9y^7$ no
8. $49p^{10}$ yes; $7p^5$

Also available online

Math Humor

Q: What did Polly Nomial call her spoiled triplets?

A: The perfect Tri Nomials.

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Learn It Online
State Resources Online

COMMON CORE

7-5 Factoring Special Products

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objectives
Factor perfect-square trinomials.

Factor the difference of two squares.

Who uses this?
Urban planners can use the area of a square park to find its length and width. (See Example 2.)

Previously, you studied the patterns of some special products of binomials. You can use those patterns to factor certain polynomials.

A trinomial is a perfect square if:
- The **first** and **last** terms are perfect squares.
- The **middle** term is two times one factor from the first term and one factor from the last term.

$$9x^2 + 12x + 4$$
$$3x \cdot 3x \quad 2(3x \cdot 2) \quad 2 \cdot 2$$

Know it! Note

Perfect-Square Trinomials

PERFECT-SQUARE TRINOMIAL	EXAMPLES
$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	$x^2 - 2x + 1 = (x - 1)(x - 1) = (x - 1)^2$

EXAMPLE 1 Recognizing and Factoring Perfect-Square Trinomials

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

A $x^2 + 12x + 36$

$$x^2 + 12x + 36$$
$$x \cdot x \quad 2(x \cdot 6) \quad 6 \cdot 6$$

The trinomial is a perfect square. Factor.

Method 1 Factor.

$x^2 + 12x + 36$

Factors of 36	Sum	
1 and 36	37	✗
2 and 18	20	✗
3 and 12	15	✗
4 and 9	13	✗
6 and 6	12	✓

$(x + 6)(x + 6)$

Method 2 Use the rule.

$x^2 + 12x + 36$ *$a = x$, $b = 6$*

$x^2 + 2(x)(6) + 6^2$ *Write the trinomial as $a^2 + 2ab + b^2$.*

$(x + 6)^2$ *Write the trinomial as $(a + b)^2$.*

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1 Introduce

EXPLORATION

Factoring Special Products

In this Exploration, you will look for patterns as you find the squares of binomials.

1. Complete the table by writing each square as a product of two factors. Then find the product of the two factors and express the result as a trinomial.

Square	Factors	Product
$(x + 2)^2$	$(x + 2)(x + 2)$	$x^2 + 4x + 4$
$(x + 3)^2$	$(x + 3)(x + 3)$	$x^2 + 6x + 9$
$(x + 5)^2$	$(x + 5)(x + 5)$	$x^2 + 10x + 25$
$(x - 4)^2$	$(x - 4)(x - 4)$	$x^2 - 8x + 16$
$(2x + 5)^2$	$(2x + 5)(2x + 5)$	$4x^2 + 20x + 25$
$(3x - 7)^2$	$(3x - 7)(3x - 7)$	$9x^2 - 42x + 49$
$(5x + 1)^2$	$(5x + 1)(5x + 1)$	$25x^2 + 10x + 1$

2. Look at the constant terms of the trinomials in your table. What do you notice? All are perfect squares.
3. Look at the x^2-terms of the trinomials in your table. What do you notice? All are perfect squares.

THINK AND DISCUSS

4. **Explain** whether it is possible for the constant term of a trinomial to be negative if the trinomial can be written as the square of a binomial.
5. **Discuss** whether you think $x^2 + 6x + 24$ can be written as the square of a binomial.

6. No; the constant term is a perfect square, so it must be positive.

Motivate

Have students write all the steps for multiplying $(2x + 3)(2x - 3)$ using the FOIL method.

$(2x)(2x) + (2x)(-3) + (3)(2x) + (3)(-3)$

$= 4x^2 - 6x + 6x - 9$

$= 4x^2 - 9$

Ask students why there is no middle term in the final step. The sum of the inner product and the outer product is zero.

Explorations and answers are provided in your online resources.

Helpful Hint

You can check your answer by using the FOIL method.

For Example 1B,
$(2x - 3)^2 =$
$(2x - 3)(2x - 3) =$
$4x^2 - 6x - 6x + 9 =$
$4x^2 - 12x + 9$

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

B $4x^2 - 12x + 9$

$4x^2 - 12x + 9$
$2x \cdot 2x \quad 2(2x \cdot 3) \quad 3 \cdot 3$ — *The trinomial is a perfect square. Factor.*

$4x^2 - 12x + 9$ — $a = 2x, b = 3$
$(2x)^2 - 2(2x)(3) + 3^2$ — $a^2 - 2ab + b^2$
$(2x - 3)^2$ — $(a - b)^2$

C $x^2 + 9x + 16$

$x^2 + 9x + 16$
$x \cdot x \quad 2(x \cdot 4) \quad 4 \cdot 4$ — $2(x \cdot 4) \neq 9x$

$x^2 + 9x + 16$ is not a perfect-square trinomial because $9x \neq 2(x \cdot 4)$.

CHECK IT OUT! **Determine whether each trinomial is a perfect square. If so, factor. If not, explain.**

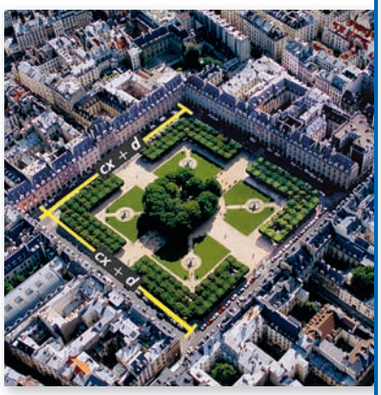
1a. $x^2 + 4x + 4$ — yes; $(x + 2)^2$

1b. $x^2 - 14x + 49$ — yes; $(x - 7)^2$

1c. $9x^2 - 6x + 4$ — no; $-6x \neq 2(3x)(2)$

EXAMPLE 2 *Problem-Solving Application*

The park in the center of the Place des Vosges in Paris, France, is in the shape of a square. The area of the park is $(25x^2 + 70x + 49)$ ft². The side length of the park is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of the park. Find the perimeter when $x = 8$ ft.



MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

1 Understand the Problem

The **answer** will be an expression for the perimeter of the park and the value of the expression when $x = 8$.

List the **important information:**

- The park is a square with area $(25x^2 + 70x + 49)$ ft².
- The side length of the park is in the form $cx + d$, where c and d are whole numbers.

2 Make a Plan

The formula for the area of a square is area = $(\text{side})^2$.

Factor $25x^2 + 70x + 49$ to find the side length of the park. Write a formula for the perimeter of the park, and evaluate the expression for $x = 8$.

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COMMON ERROR ALERT

When checking the answer for **Example 1A,** students might multiply $(x + 6)^2$ as $x^2 + 6^2 = x^2 + 36$. Encourage students to write $(x + 6)^2$ as $(x + 6)(x + 6)$ and then use the FOIL method.

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Additional Examples

Example 1

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

A. $9x^2 - 15x + 64$
no; $-15x \neq 2(3x \cdot 8)$

B. $81x^2 + 90x + 25$
yes; $(9x + 5)^2$

C. $36x^2 - 10x + 14$
No; 14 is not a perfect square.

Example 2

A square piece of cloth must be cut to make a tablecloth. The area needed is $(16x^2 - 24x + 9)$ in². The dimensions of the cloth are of the form $cx - d$, where c and d are whole numbers. Find an expression for the perimeter of the cloth. Find the perimeter when $x = 11$ inches.
$16x - 12$; 164 in.

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- Identify the steps for determining whether a trinomial is a perfect-square trinomial.
- When factoring a perfect-square trinomial, do you think it is easier to list the factors or use the pattern? Explain your reasoning.

EXAMPLE 2

- What must be true about the factors of a rectangular area for that area to be a square? Explain.

2 Teach

Guided Instruction

Before beginning this lesson, review factoring trinomials and identifying perfect squares. Point out that perfect-square trinomials can be factored using the methods from the previous sections. Because these polynomials contain perfect-square numbers, easier methods are shown. Emphasize that factoring the difference of two squares is possible only for the difference. The sum of two squares cannot be factored.

Reaching All Learners *Through Number Sense*

Show students a quick way to rule out the possibility of a trinomial being a perfect-square trinomial. Remind them that if $ax^2 + bx + c$ is a perfect-square trinomial, then b must be an even number. This is because in a perfect-square trinomial, $b = 2 \cdot \sqrt{a} \cdot \sqrt{c}$. If b is even, then the trinomial *might* be a perfect square.

Teaching Tip **Inclusion** After completing Step 3 of **Example 2,** show that the numerical value of the perimeter could be found by first substituting $x = 8$ into $5x + 7$ to find the length of one side. The length of one side is 47 ft, and the perimeter is 47 × 4, or 188, ft.

3 Solve

$25x^2 + 70x + 49$ — $a = 5x, b = 7$

$(5x)^2 + 2(5x)(7) + 7^2$ — *Write the trinomial as $a^2 + 2ab + b^2$.*

$(5x + 7)^2$ — *Write the trinomial as $(a + b)^2$.*

$25x^2 + 70x + 49 = (5x + 7)(5x + 7)$

The side length of the park is $(5x + 7)$ ft.

Write a formula for the perimeter of the park.

$P = 4s$ — *Write the formula for the perimeter of a square.*

$= 4(5x + 7)$ — *Substitute the side length for s.*

$= 20x + 28$ — *Distribute 4.*

An expression for the perimeter of the park in feet is $20x + 28$.

Evaluate the expression when $x = 8$.

$P = 20x + 28$

$= 20(8) + 28$ — *Substitute 8 for x.*

$= 188$

When $x = 8$ ft, the perimeter of the park is 188 ft.

4 Look Back

For a square with a perimeter of 188 ft, the side length is $\frac{188}{4} = 47$ ft and the area is $47^2 = 2209$ ft^2.

Evaluate $25x^2 + 70x + 49$ for $x = 8$:

$25(8)^2 + 70(8) + 49$

$1600 + 560 + 49$

2209 ✓

CHECK IT OUT! **2. What if...?** A company produces square sheets of aluminum, each of which has an area of $(9x^2 + 6x + 1)$ m^2. The side length of each sheet is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of a sheet. Find the perimeter when $x = 3$ m. **$4(3x + 1)$ m; 40 m**

Previously, you learned that the difference of two squares has the form $a^2 - b^2$. The difference of two squares can be written as the product $(a + b)(a - b)$. You can use this pattern to factor some polynomials.

A polynomial is a difference of two squares if:

- There are two terms, one subtracted from the other.
- Both terms are perfect squares.

$4x^2 - 9$

$2x \cdot 2x \quad 3 \cdot 3$

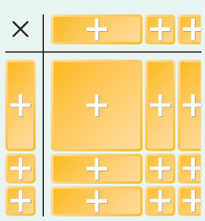
Know it! Note

Difference of Two Squares

DIFFERENCE OF TWO SQUARES	EXAMPLE
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$

Reaching All Learners Through Modeling

Recall modeling the factors of trinomials with algebra tiles (MK). The tiles must be arranged in a rectangle. When the factors of a perfect-square trinomial are modeled with algebra tiles, the tiles will be arranged in a square.



EXAMPLE 3 Recognizing and Factoring the Difference of Two Squares

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

Reading Math

Recognize a difference of two squares: the coefficients of variable terms are perfect squares, powers on variable terms are even, and constants are perfect squares.

A $x^2 - 81$

$x^2 - 81$ (square roots: $x \cdot x$ and $9 \cdot 9$) — *The polynomial is a difference of two squares.*

$x^2 - 9^2$ — *$a = x$, $b = 9$*

$(x + 9)(x - 9)$ — *Write the polynomial as $(a + b)(a - b)$.*

$x^2 - 81 = (x + 9)(x - 9)$

B $9p^4 - 16q^2$

$9p^4 - 16q^2$ ($3p^2 \cdot 3p^2$ and $4q \cdot 4q$) — *The polynomial is a difference of two squares.*

$(3p^2)^2 - (4q)^2$ — *$a = 3p^2$, $b = 4q$*

$(3p^2 + 4q)(3p^2 - 4q)$ — *Write the polynomial as $(a + b)(a - b)$.*

$9p^4 - 16q^2 = (3p^2 + 4q)(3p^2 - 4q)$

C $x^6 - 7y^2$

$x^6 - 7y^2$ ($x^3 \cdot x^3$) — *$7y^2$ is not a perfect square.*

$x^6 - 7y^2$ is not the difference of two squares because $7y^2$ is not a perfect square.

CHECK IT OUT! **Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.**

3a. $1 - 4x^2$ — yes; $(1 - 2x)(1 + 2x)$

3b. $p^8 - 49q^6$ — yes; $(p^4 + 7q^3)(p^4 - 7q^3)$

3c. $16x^2 - 4y^5$ — No; $4y^5$ is not a perfect square.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. The binomial $1 - x^4$ is a difference of two squares. Use the rule to identify a and b in $1 - x^4$.
2. The polynomial $x^2 + 8x + 16$ is a perfect-square trinomial. Use the rule to identify a and b in $x^2 + 8x + 16$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write an example of each type of special product and factor it.

Know it! Note

Special Product	Factored Form
Perfect-square trinomial with positive coefficient of middle term	
Perfect-square trinomial with negative coefficient of middle term	
Difference of two squares	

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Additional Examples

Example 3

Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.

A. $3p^2 - 9q^4$ No; $3p^2$ is not a perfect square.

B. $100x^2 - 4y^2$ yes; $(10x + 2y)(10x - 2y)$

C. $x^4 - 25y^6$ yes; $(x^2 + 5y^3)(x^2 - 5y^3)$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- Explain why the product of binomials of the form $(a + b)(a - b)$ is not a trinomial.

Teaching Tip **Math Background** In **Example 3C,** students might think that x^6 is not a perfect square because the number 6 is not a perfect square. Remind them of the Product of Powers Property, which states that $a^m \cdot a^n = a^{m+n}$. Thus, x^6 is a perfect square because it can be written as $x^3 \cdot x^3$.

3 Close

Summarize

Have students name the characteristics of

- a perfect-square trinomial of the form $ax^2 + bx + c$. 1) a and c must be perfect squares. 2) $b = 2\sqrt{ac}$
- a difference of squares of the form $a^2 - b^2$. 1) Both terms must be perfect squares. 2) It must be a difference.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. $a = 1$; $b = x^2$
2. $a = x$; $b = 4$
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 14–20, 27–29
- **Average** 14–20, 27, 29, 38
- **Advanced** 14–20, 27, 29, 38

If you finished Examples **1–3**
- **Basic** 14–36, 38–49
- **Average** 14–40, 46–50
- **Advanced** 14–29, 30–34 even, 36–40, 47–54

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 16, 20, 22, 26, 38

Answers

14. yes; $(2x - 1)^2$
15. No; the last term must be pos.
16. yes; $(6x - 1)^2$
17. no; $10x \neq 2(5x)(2)$
18. yes; $(3x + 3)^2$
19. yes; $(4x - 5)^2$
30. $(x - 4)^2$; perf.-square trinomial
31. $(10x + 9y)(10x - 9y)$; diff. of 2 squares
32. $(6x + 2)^2$; perf.-square trinomial
33. $(2r^3 + 5s^3)(2r^3 - 5s^3)$; diff. of 2 squares
34. $(7x - 5)^2$; perf.-square trinomial
35. $(x^7 + 12)(x^7 - 12)$; diff. of 2 squares
36–37. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 42, 48, 59, 64

Construct viable arguments and critique the reasoning of others. Exercises 43, 49, 51

Attend to precision. Exercise 50

Look for and express regularity in repeated reasoning. Exercises 52–54

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State Resources Online

7-5 Exercises

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 - 4x + 4$ yes; $(x - 2)^2$
2. $x^2 - 4x - 4$ No; the last term must be pos.
3. $9x^2 - 12x + 4$ yes; $(3x - 2)^2$
4. $x^2 + 2x + 1$ yes; $(x + 1)^2$
5. $x^2 - 6x + 9$ yes; $(x - 3)^2$
6. $x^2 - 6x - 9$ No; the last term must be pos.

SEE EXAMPLE 2

7. **City Planning** A city purchases a square plot of land with an area of $(x^2 + 24x + 144)$ yd^2 for a park. The dimensions of the plot are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the park. Find the perimeter when $x = 10$ yd. $4(x + 12)$; 88 yd

SEE EXAMPLE 3

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

8. $1 - 4x^2$ yes; $(1 + 2x)(1 - 2x)$
9. $s^2 - 4^2$ yes; $(s + 4)(s - 4)$
10. $81x^2 - 1$ yes; $(9x + 1)(9x - 1)$
11. $4x^4 - 9y^2$ yes; $(2x^2 + 3y)(2x^2 - 3y)$
12. $x^8 - 50$ No; 50 is not a perf. square.
13. $x^6 - 9$ yes; $(x^3 + 3)(x^3 - 3)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–19	1
20	2
21–26	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Determine whether the trinomial is a perfect square. If so, factor. If not, explain.

14. $4x^2 - 4x + 1$
15. $4x^2 - 4x - 1$
16. $36x^2 - 12x + 1$
17. $25x^2 + 10x + 4$
18. $9x^2 + 18x + 9$
19. $16x^2 - 40x + 25$

20. **Measurement** You are given a sheet of paper and told to cut out a square piece with an area of $(4x^2 - 44x + 121)$ mm^2. The dimensions of the square have the form $ax - b$, where a and b are whole numbers. Find an expression for the perimeter of the square you cut out. Find the perimeter when $x = 41$ mm. $4(2x - 11)$; 284 mm

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

21. $1^2 - 4x^2$ yes; $(1 + 2x)(1 - 2x)$
22. $25m^2 - 16n^2$ yes; $(5m - 4n)(5m + 4n)$
23. $4x - 9y$ No; $4x$ and $9y$ are not perf. squares.
24. $49p^{12} - 9q^6$ yes; $(7p^6 - 3q^3)(7p^6 + 3q^3)$
25. $9^2 - 100x^4$ yes; $(9 - 10x^2)(9 + 10x^2)$
26. $x^3 - y^3$ No; x^3 and y^3 are not perf. squares.

Find the missing term in each perfect-square trinomial.

27. $x^2 + 14x +$ 49
28. $9x^2 +$ 30x $+ 25$
29. 4y^2 $- 36y + 81$

Factor each polynomial using the rule for perfect-square trinomials or the rule for a difference of two squares. Tell which rule you used.

30. $x^2 - 8x + 16$
31. $100x^2 - 81y^2$
32. $36x^2 + 24x + 4$
33. $4r^6 - 25s^6$
34. $49x^2 - 70x + 25$
35. $x^{14} - 144$

36. **Write About It** What is similar about a perfect-square trinomial and a difference of two squares? What is different?

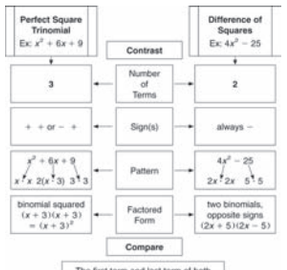
37. **Critical Thinking** Describe two ways to create a perfect-square trinomial.

38. For what value of b would $(x + b)(x + b)$ be the factored form of $x^2 - 22x + 121$? $b = -11$

39. For what value of c are the factors of $x^2 + cx + 256$ the same? $c = 32$

READING STRATEGIES

The chart below shows how to recognize and factor two special products.



Identify each of the following as a perfect square trinomial (P), a difference of squares (D), or neither (N).

1. $x^2 - 16$ D
2. $x^2 - 14x + 49$ P
3. $100x^2 - 36$ D
4. $4x^2 - 12x - 9$ N
5. $x^2 + 14x + 49$ P
6. $25x^2 + 64$ N

Factor each special product.

7. $16x^2 - 9$ $(4x + 3)(4x - 3)$
8. $121x^2 - 44x + 4$ $(11x - 2)^2$
9. $81x^2 - 1$ $(9x + 1)(9x - 1)$

RETEACH

If a polynomial is a perfect square trinomial, the polynomial can be factored using a pattern.

$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

Determine whether $4x^2 + 20x + 25$ is a perfect square trinomial. If so, factor it. If not, explain why.

Step 1: Find a, b, then $2ab$.

$a = \sqrt{4x^2} = 2x$ — *The first term is a perfect square.*
$b = \sqrt{25} = 5$ — *The last term is a perfect square.*
$2ab = 2(2x)(5) = 20x$ — *Middle term* $(20x) = 2ab$.

Therefore, $4x^2 + 20x + 25$ is a perfect square trinomial.

Step 2: Substitute expressions for a and b into $(a + b)^2$.

$(2x + 5)^2$

Determine whether $9x^2 + 25x + 36$ is a perfect square trinomial. If so, factor it. If not, explain why.

Step 1: Find a, b, then $2ab$.

$a = \sqrt{9x^2} = 3x$ — *The first term is a perfect square.*
$b = \sqrt{36} = 6$ — *The last term is a perfect square.*
$2ab = 2(3x)(6) = 36x$ — *Middle term* $(25x) \neq 2ab$.

STOP
Because $25x$ does not equal $2ab$, $9x^2 - 25x + 36$ is not a perfect square trinomial.

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $9x^2 + 30x + 100$
   $a = 3x$
   $b = 10$
   $2ab = 60x$
   Factor or explain: $30x \neq 2ab$
2. $x^2 - 14x + 49$
   $a = x$
   $b = 7$
   $2ab = 14x$
   Factor or explain: $(x - 7)^2$
3. $25x^2 + 20x + 4$
   $a = 5x$
   $b = 2$
   $2ab = 20x$
   Factor or explain: $(5x + 2)^2$

MULTI-STEP TEST PREP

40. Juanita designed a vegetable garden in the shape of a square and purchased fencing for that design. Then she decided to change the design to a rectangle. $(x-5)(x+5)$

a. The square garden had an area of x^2 ft^2. The area of the rectangular garden is (x^2-25) ft^2. Factor this expression.

b. The rectangular garden must have the same perimeter as the square garden, so Juanita added a number of feet to the length and subtracted the same number of feet from the width. Use your factors from part **a** to determine how many feet were added to the length and subtracted from the width.

c. If the original length of the square garden was 8 feet, what are the length and width of the new garden? $\ell = 13$ ft; $w = 3$ ft

40b. ℓ: added 5 ft; w: subtracted 5 ft

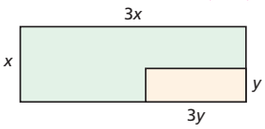
41. Multi-Step The area of a square is represented by $25z^2 - 40z + 16$.

a. What expression represents the length of a side of the square? $5z - 4$

b. What expression represents the perimeter of the square? $20z - 16$

c. What are the length of a side, the perimeter, and the area of the square when $z = 3$? 11; 44; 121

42. Multi-Step A small rectangle is drawn inside a larger rectangle as shown.



a. What is the area of each rectangle? $3x^2$; $3y^2$

b. What is the area of the green region? $3x^2 - 3y^2$

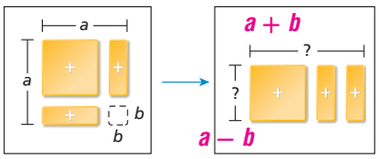
c. Factor the expression for the area of the green region. (*Hint:* First factor out the common factor of 3 and then factor the binomial.) $3(x+y)(x-y)$

43. Evaluate each expression for the values of x.

	x	$x^2 + 10x + 25$	$(x+5)^2$	$(x-5)^2$	$x^2 - 10x + 25$	$x^2 - 25$
a.	−5	0	0	100	100	0
b.	−1	16	16	36	36	−24
c.	0	25	25	25	25	−25
d.	1	36	36	16	16	−24
e.	5	100	100	0	0	0

44. In the table above, which columns have equivalent values? Explain why.

45. Geometry A model for the difference of two squares is shown below. Copy and complete the second figure by writing the missing labels.



46. ERROR ANALYSIS Two students factored $25x^4 - 9y^2$. Which is incorrect? Explain the error.

Student A; $(5x)(5x) \neq 25x^4$ and $(-3)(3) \neq 9y^2$.

A

$25x^4 - 9y^2$

$(5x - 3)(5x + 3)$

B

$25x^4 - 9y^2$

$(5x^2 - 3y)(5x^2 + 3y)$

© Stockbyte

MULTI-STEP TEST PREP **Exercise 40** involves using factoring to find dimensions of squares and rectangles. This exercise prepares students for the Multi-Step Test Prep.

Answers

44. The first and second columns and the third and fourth columns have the same values; the second column is the factored form of the first column. The third column is the factored form of the fourth column.

PRACTICE A

PRACTICE C

PRACTICE B

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $x^2 + 6x + 9$
   yes; $(x+3)^2$
2. $4x^2 + 20x + 25$
   yes; $(2x+5)^2$
3. $36x^2 - 24x + 16$
   no; $24x \neq 2(6x \square 4)$
4. $9x^2 - 12x + 4$
   yes; $(3x-2)^2$
5. A rectangular fountain in the center of a shopping mall has an area of $(4x^2 + 12x + 9)$ ft^2. The dimensions of the fountain are of the form $cx + d$, where c and d are whole numbers. Find an expression for the perimeter of the fountain. Find the perimeter when $x = 2$ ft.
   $4(2x+3)$ ft; 28 ft

Determine whether each binomial is the difference of perfect squares. If so, factor it. If not, explain why.

6. $x^2 - 16$
   yes; $(x+4)(x-4)$
7. $9b^4 - 200$
   no; 200 is not a perfect square.
8. $1 - m^2$
   yes; $(1+m^2)(1-m^2)$
9. $36s^2 - 4t^2$
   yes; $(6s+2t)(6s-2t)$
10. $x^2y^2 + 196$
    no; the operation between the two squares is addition.

PROBLEM SOLVING

Write the correct answer.

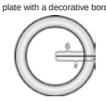
1. A rectangular fountain has an area of $(16x^2 + 8x + 1)$ ft^2. The dimensions of the rectangle have the form $ax + b$, where a and b are whole numbers. Write an expression for the perimeter of the fountain. Then find the perimeter when $x = 2$ feet.
   $16x + 4$; 36 feet

2. A square tabletop has an area of $(9x^2 - 90x + 225)$ cm^2. The dimensions of the tabletop have the form $cx - d$, where c and d are whole numbers. Write an expression for the perimeter of the tabletop. Then find the perimeter when $x = 25$ centimeters.
   $12x - 60$; 240 cm

3. The floor plan of a daycare center is shown.

   The arts and crafts area in the lower right corner is not carpeted. The rest of the center is carpeted. Write an expression, in factored form, for the area of the floor that is carpeted.
   $(x+2y)(x-2y)$

4. A plate with a decorative border is shown.

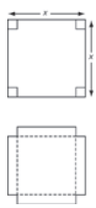


   Write an expression, in factored form, for the area of the border. (*Hint:* First factor out the GCF.)
   $4(x+6)(x-6)$

Nelson is making open top boxes by cutting out corners from a sheet of cardboard, folding the edges up, and then taping them together. Select the best answer.

5. Nelson cut corners so that each corner was a square with side lengths of 4. What is the total area of the remaining piece of cardboard?
   A $x^2 - 8x + 16$ C $x^2 - 16x + 64$
   B $x^2 + 8x + 16$ D $x^2 + 16x + 64$

6. What are the dimensions of the square corners if the total remaining area is $x^2 - 4x + 4$?
   F 1 by 1 H 4 by 4
   G 2 by 2 J 8 by 8



CHALLENGE

You already know the facts stated below.

$a^2 - b^2 = (a+b)(a-b)$ and $a^2 + b^2$ cannot be factored by using real numbers.

In the exercises that follow, you will be able to study the factorability of differences and sums involving exponents greater than or equal to 2.

In Exercises 1 and 2, consider $a^n - b^n$ for $n = 3$ and $n = 4$, respectively.

1. Show that $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ by performing the multiplication started at right. Show your work in the space provided.

$$\begin{array}{r} a^2 + ab + b^2 \\ \times \quad a - b \\ \hline -ba^2 - ab^2 - b^3 \\ +a^3 + ba^2 + ab^2 \\ \hline a^3 \qquad -b^3 \end{array}$$

2. Show that $a^4 - b^4 = (a+b)(a-b)(a^2+b^2)$ by following the reasoning started at right. Notice that the difference of fourth powers is written as a new difference of squares.

$$a^4 - b^4 = (a^2)^2 - (b^2)^2$$
$$= (\boxed{a^2} + \boxed{b^2})(\boxed{a^2} - \boxed{b^2})$$
$$= (a^2 + b^2)(a+b)(a-b)$$

In Exercises 3 and 4, consider $a^n + b^n$ for $n = 3$ and $n = 2$, respectively.

3. Show that $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ by performing the multiplication started at right. Show your work in the space provided.

$$\begin{array}{r} a^2 - ab + b^2 \\ \times \quad a + b \\ \hline ba^2 - ab^2 + b^3 \\ +a^3 - ba^2 + ab^2 \\ \hline a^3 \qquad +b^3 \end{array}$$

4. Suppose that $a^2 + b^2$ can be factored as $a^2 + b^2 = (ra + sb)(ta + bu)$, where r, s, t, and u are numbers and a and b are variables. The reasoning shown at right will help you see that $a^2 + b^2$ cannot be factored.

$$\begin{aligned} a^2 + b^2 &= (ra + sb)(ta + bu) \\ &= (rt)\,a^2 + (st + ru)\,ab + (su)b^2 \end{aligned}$$
So, $1 = rt$, $0 = st + ru$, and $1 = su$
$r = \frac{1}{t}$, $-st = ru$, and $s = \frac{1}{u}$
Thus, $-\frac{t}{u} = \frac{u}{t}$
$-t^2 = u^2$
$t = 0$ and $u = 0$

   a. How do you know that $1 = rt$, $0 = st + ru$, and $1 = su$?
   If $t = 0$ and $u = 0$, then r and s are given by undefined expressions. Thus, there are no numbers r, s, t, and u for which $a^2 + b^2$ can be factored as $(ra + sb)(ta + bu)$.

   b. Explain how $t = 0$ and $u = 0$ tell you that there are no values of r and s that make the factorization possible.
   If two polynomials are equal, their corresponding coefficients are equal. The coefficients of a^2, ab, and b^2 are equal.

TEST PREP DOCTOR For **Exercise 47,** encourage students to write each expression in factored form before substituting values for x and y.

In **Exercise 48,** the trinomial contains only positive numbers. Therefore, the factored answer must contain only positive numbers, so **F, G,** and **H** can be eliminated.

Journal

Explain how you can find the missing term in the perfect-square trinomial $100x^2 + 120x + \square$.

ALTERNATIVE ASSESSMENT

Create one trinomial that is a perfect-square trinomial and one that is not. Factor the first trinomial. Explain why the second trinomial is not a perfect square. Repeat for the difference of two squares.

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7-5 Lesson Quiz

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $64x^2 - 40x + 25$
   no; $-40x \neq 2(8x \cdot 5)$
2. $121x^2 - 44x + 4$
   yes; $(11x - 2)^2$
3. $49x^2 + 140x + 100$
   yes; $(7x^2 + 10)^2$
4. A fence will be built around a square garden with an area of $(49x^2 + 56x + 16)$ ft². The dimensions of the garden are in the form $cx + d$, where c and d are whole numbers. Find an expression for the perimeter of the garden. Find the perimeter when $x = 5$ feet.
   $P = 28x + 16$; 156 ft

Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.

5. $9x^2 - 144y^4$
   yes; $(3x + 12y^2)(3x - 12y^2)$
6. $30x^2 - 64y^2$ No; $30x^2$ is not a perfect square.
7. $121x^2 - 4y^8$
   yes; $(11x + 2y^4)(11x - 2y^4)$

Also available online

TEST PREP

47. A polynomial expression is evaluated for the x- and y-values shown in the table. Which expression could have been evaluated to give the values shown in the third column?

x	y	Value of Expression
0	0	0
−1	−1	0
1	1	0
1	−1	4

Ⓐ $x^2 - y^2$
Ⓑ $x^2 + 2xy + y^2$
Ⓒ $x^2 - 2xy + y^2$
Ⓓ None of the above

48. The area of a square is $4x^2 + 20x + 25$. Which expression can also be used to model the area of the square?

Ⓕ $(2x - 5)(5 - 2x)$
Ⓖ $(2x + 5)(2x - 5)$
Ⓗ $(2x - 5)^2$
Ⓙ $(2x + 5)^2$

49. Gridded Response Evaluate the polynomial expression $x^2 - 18x + 81$ for $x = 10$. **1**

CHALLENGE AND EXTEND

50. The binomial $81x^4 - 16$ can be factored using the rule for a difference of two squares.

a. Fill in the factorization: $81x^4 - 16$
$(9x^2 + \square)(\square - \square)$ **4; $9x^2$; 4**

b. One binomial from part **a** can be further factored. Identify the binomial and factor it. $9x^2 - 4 = (3x + 2)(3x - 2)$

c. Write your own binomial that can be factored twice as the difference of two squares. **Possible answer:** $x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x + 1)(x - 1)$

51. The expression $4 - (v + 2)^2$ is the difference of two squares, because it fits the rule $a^2 - b^2$.

a. Identify a and b in the expression. $a = 2$; $b = v + 2$

b. Factor and simplify $4 - (v + 2)^2$.
$[2 + (v + 2)][2 - (v + 2)] = (v + 4)(-v) = -v^2 - 4v$

The *difference of cubes* is an expression of the form $a^3 - b^3$. It can be factored according to the rule $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. For each binomial, identify a and b, and factor using the rule.

52. $x^3 - 1$
$a = x$; $b = 1$;
$(x - 1)(x^2 + x + 1)$

53. $27y^3 - 64$
$a = 3y$; $b = 4$;
$(3y - 4)(9y^2 + 12y + 16)$

54. $n^6 - 8$
$a = n^2$; $b = 2$;
$(n^2 - 2)(n^4 + 2n + 4)$

Mental Math

Recognizing patterns of special products can help you perform multiplication mentally.

Remember these special products.

Patterns of Special Products	
Difference of Two Squares	$(a + b)(a - b) = a^2 - b^2$
Perfect-Square Trinomial	$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$

Example 1

Simplify $17^2 - 7^2$.

This expression is a difference of two squares with $a = 17$ and $b = 7$.

$a^2 - b^2 = (a + b)(a - b)$ *Write the rule for a difference of two squares.*

$17^2 - 7^2 = (17 + 7)(17 - 7)$ *Substitute 17 for a and 7 for b.*

$= (24)(10)$ *Simplify.*

$= 240$

Example 2

Simplify $14^2 + 2(14)(6) + 6^2$.

This expression is a perfect-square trinomial with $a = 14$ and $b = 6$.

$a^2 + 2ab + b^2 = (a + b)^2$ *Write the rule for a perfect-square trinomial.*

$14^2 + 2(14)(6) + 6^2 = (14 + 6)^2$ *Substitute 14 for a and 6 for b.*

$= (20)^2$ *Simplify.*

$= 400$

Try This

Simplify each expression using the rules for special products.

1. $18^2 - 12^2$
2. $11^2 + 2(11)(14) + 14^2$
3. $22^2 - 18^2$
4. $38^2 - 2(38)(27) + 27^2$
5. $29^2 - 2(29)(17) + 17^2$
6. $55^2 + 2(55)(45) + 45^2$
7. $14^2 - 9^2$
8. $13^2 - 12^2$
9. $14^2 + 2(14)(16) + 16^2$

Answers to *Try This*

1. $(18 + 12)(18 - 12) = 30(6) = 180$
2. $(11 + 14)^2 = 25^2 = 625$
3. $(22 + 18)(22 - 18) = 40(4) = 160$
4. $(38 - 27)^2 = 11^2 = 121$
5. $(29 - 17)^2 = 12^2 = 144$
6. $(55 + 45)^2 = 100^2 = 10{,}000$
7. $(14 + 9)(14 - 9) = 23(5) = 115$
8. $(13 + 12)(13 - 12) = 25(1) = 25$
9. $(14 + 16)^2 = 30^2 = 900$

Organizer

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Apply patterns of special products to performing multiplication mentally.

Online Edition

Teach

Remember

Students review and apply mental math techniques.

Close

Assess

Have students explain how to use patterns of special products and mental math to simplify Try This **Problems 4** and **8.**

Learn It Online
State Resources Online

7-6 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Choose an appropriate method for factoring a polynomial.

Combine methods for factoring a polynomial.

Online Edition
Tutorial Videos

Countdown

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Warm Up

Factor each trinomial.

1. $x^2 + 13x + 40$
   $(x + 5)(x + 8)$
2. $5x^2 - 18x - 8$
   $(5x + 2)(x - 4)$
3. Factor the perfect-square trinomial $16x^2 + 40x + 25$.
   $(4x + 5)(4x + 5)$
4. Factor $9x^2 - 25y^2$ using the difference of two squares.
   $(3x + 5y)(3x - 5y)$

Also available online

Math Humor

Q: Where do math teachers buy their products?

A: From the factor-*y*.

COMMON CORE

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Learn It Online
State Resources Online

COMMON CORE

7-6

Choosing a Factoring Method

CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

Objectives
Choose an appropriate method for factoring a polynomial.

Combine methods for factoring a polynomial.

Why learn this?
You will need to factor polynomials to solve quadratic equations, which have many applications in physics. (See Exercise 42.)

The height of a leaping ballet dancer can be modeled by a quadratic polynomial. Solving an equation that contains that polynomial may require factoring the polynomial.

Recall that a polynomial is in its fully factored form when it is written as a product that cannot be factored further.

EXAMPLE 1 **Determining Whether an Expression Is Completely Factored**

Tell whether each expression is completely factored. If not, factor it.

A $2x(x^2 + 4)$

$2x(x^2 + 4)$ *Neither 2x nor $x^2 + 4$ can be factored further.*

$2x(x^2 + 4)$ is completely factored.

B $(2x + 6)(x + 5)$

$(2x + 6)(x + 5)$ *2x + 6 can be further factored.*

$2(x + 3)(x + 5)$ *Factor out 2, the GCF of 2x and 6.*

$2(x + 3)(x + 5)$ is completely factored.

Caution!
$x^2 + 4$ is a *sum* of squares, and cannot be factored.

CHECK IT OUT! **Tell whether each expression is completely factored. If not, factor it.**

1a. $5x^2(x - 1)$ **yes**

1b. $(4x + 4)(x + 1)$ **no; $4(x + 1)^2$**

To factor a polynomial completely, you may need to use more than one factoring method. Use the steps below to factor a polynomial completely.

Factoring Polynomials	
Step 1	Check for a greatest common factor.
Step 2	Check for a pattern that fits the difference of two squares or a perfect-square trinomial.
Step 3	To factor $x^2 + bx + c$, look for two numbers whose sum is *b* and whose product is *c*. To factor $ax^2 + bx + c$, check factors of *a* and factors of *c* in the binomial factors. The sum of the products of the outer and inner terms should be *b*.
Step 4	Check for common factors.

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1 Introduce

EXPLORATION

Choosing a Factoring Method

You now know several methods for factoring polynomials. This Exploration will help you see how some of the methods can be used in combination with each other.

1. Factor out the GCF of each polynomial in the table. Write the result in the middle column of the table. Then look at the remaining factor. Identify its type and write this in the right-hand column of the table.

Polynomial	Factor Out the GCF	Type of Factor That Remains
$2x^3 + 12x^2 + 18x$	$2x(x^2 + 6x + 9)$	Perfect-square trinomial
$12a^3b + 60a^2b + 75ab$	$3ab(4a^2 + 20a + 25)$	Perfect-square trinomial
$x^5 - 36x^3$	$x^3(x^2 - 36)$	Difference of squares
$2xy^3 - 20xy^2 + 50xy$	$2xy(y^2 - 10y + 25)$	Perfect-square trinomial
$3y^2 - 48y$	$3y(y^2 - 16)$	Difference of squares

2. Explain what you would do to continue factoring $2x^3 + 12x^2 + 18x$ after factoring out the GCF, 2*x*.
3. Explain what you would do to continue factoring $x^5 - 36x^3$ after factoring out the GCF. Write $x^3 - 36$ as $(x - 6)(x + 6)$.

THINK AND DISCUSS

4. **Describe** the advantages of first factoring out the GCF when factoring a polynomial.
5. **Explain** how factoring out the GCF can be helpful even if the remaining factor is not one of the types that you found in the table.

Motivate

Have students describe characteristics of the following polynomials. Possible answers:

$9x^2 - 25y^2$ first and last terms are perfect squares; difference of two terms

$3x + 6$ common factor in each term; binomial

$2x^3 + 6x^2 - 5x - 15$ four terms; common factor in first two terms; common factor in last two terms.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Factoring by GCF and Recognizing Patterns

Factor $-2xy^2 + 16xy - 32x$ completely. Check your answer.

$-2xy^2 + 16xy - 32x$

$-2x(y^2 - 8y + 16)$ *Factor out the GCF. $y^2 - 8y + 16$ is a perfect-square trinomial of the form $a^2 - 2ab + b^2$.*

$-2x(y - 4)^2$ *$a = y$, $b = 4$*

Check $-2x(y - 4)^2 = -2x(y^2 - 8y + 16)$

$= -2xy^2 + 16xy - 32x$ ✓

CHECK IT OUT! **Factor each polynomial completely. Check your answer.**

2a. $4x^3 + 16x^2 + 16x$ $4x(x + 2)^2$

2b. $2x^2y - 2y^3$ $2y(x - y)(x + y)$

If none of the factoring methods work, the polynomial is unfactorable.

EXAMPLE 3 Factoring by Multiple Methods

Factor each polynomial completely.

A $2x^2 + 5x + 4$

$2x^2 + 5x + 4$

$(\square x + \square)(\square x + \square)$ *The GCF is 1 and there is no pattern. $a = 2$ and $c = 4$; Outer + Inner = 5*

Factors of 2	Factors of 4	Outer + Inner	
1 and 2	1 and 4	$1(4) + 2(1) = 6$	✗
1 and 2	4 and 1	$1(1) + 2(4) = 9$	✗
1 and 2	2 and 2	$1(2) + 2(2) = 6$	✗

$2x^2 + 5x + 4$ is unfactorable.

B $3n^4 - 15n^3 + 12n^2$

$3n^2(n^2 - 5n + 4)$ *Factor out the GCF. There is no pattern. $b = -5$ and $c = 4$; look for factors of 4 whose sum is -5.*

$(n + \square)(n + \square)$

Factors of 4	Sum	
-1 and -4	-5	✓

The factors needed are -1 and -4.

$3n^2(n - 1)(n - 4)$

C $4x^3 + 18x^2 + 20x$

$2x(2x^2 + 9x + 10)$ *Factor out the GCF. There is no pattern. $a = 2$ and $c = 10$; Outer + Inner = 9*

$(\square x + \square)(\square x + \square)$

Factors of 2	Factors of 10	Outer + Inner	
1 and 2	1 and 10	$1(10) + 2(1) = 12$	✗
1 and 2	10 and 1	$1(11) + 2(10) = 21$	✗
1 and 2	2 and 5	$1(5) + 2(2) = 9$	✓

$(x + 2)(2x + 5)$

$2x(x + 2)(2x + 5)$

Remember!

For a polynomial of the form $ax^2 + bx + c$, if there are no integers whose sum is b and whose product is ac, then the polynomial is unfactorable.

COMMON ERROR ALERT

Students often think the factoring is complete after using one factoring method. Remind students to continue to look for common factors and factoring patterns until the polynomial is completely factored.

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Additional Examples

Example 1

Tell whether each expression is completely factored. If not, factor it.

A. $3x^2(6x - 4)$ no; $6x^2(3x - 2)$

B. $(x^2 + 1)(x - 4)$ completely factored

Example 2

Factor each polynomial completely. Check your answer.

A. $10x^2 + 48x + 32$ $2(5x + 4)(x + 4)$

B. $8x^6y^2 - 18x^2y^2$ $2x^2y^2(2x^2 - 3)(2x^2 + 3)$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- How do you check that a polynomial has been completely factored?

EXAMPLE 2

- What are the benefits of looking for the GCF first when factoring a polynomial?

2 Teach

Guided Instruction

Be sure that students are comfortable with all methods of factoring before beginning this section. Encourage them to first look for the GCF, then look for a pattern (for difference of squares or perfect-square trinomial), and lastly begin checking factors for a and c. Remind students that some polynomials cannot be factored. Have students check their answers to determine whether they have factored correctly.

Reaching All Learners

Through Graphic Organizers

Have groups of students create flowcharts for factoring polynomials. Include:

- Look for GCF.
- If binomial, then difference of squares OR cannot factor
- If trinomial, then perfect-square trinomial OR find factors of a and c OR cannot factor.
- If four or more terms, then factor by grouping OR cannot factor.
- Can it be factored more?

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Additional Examples

Example 3

Factor each polynomial completely.

A. $9x^2 + 3x - 2$
$(3x - 1)(3x + 2)$

B. $12b^3 + 48b^2 + 48b$
$12b(b + 2)^2$

C. $4y^2 + 12y - 72$
$4(y + 6)(y - 3)$

D. $(x^4 - x^2)$
$x^2(x + 1)(x - 1)$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- For what type of polynomials do you consider factoring by grouping?
- What methods of factoring do you consider when factoring a trinomial?

Teaching Tip Math Background

Remind students that because of the Commutative Property of Multiplication, $(a + b)(a - b) = (a - b)(a + b)$. Thus the factors can be written in any order.

D $p^5 - p$

$p(p^4 - 1)$ *Factor out the GCF.*

$p(p^2 + 1)(p^2 - 1)$ *$p^4 - 1$ is a difference of two squares.*

$p(p^2 + 1)(p + 1)(p - 1)$ *$p^2 - 1$ is a difference of two squares.*

CHECK IT OUT! Factor each polynomial completely. Check your answer.

3a. $3x^2 + 7x + 4$

3b. $2p^5 + 10p^4 - 12p^3$

3c. $9q^6 + 30q^5 + 24q^4$

3d. $2x^4 + 18$

3a. $(3x + 4)(x + 1)$

3b. $2p^3(p + 6)(p - 1)$

3c. $3q^4(3q + 4)(q + 2)$

3d. $2(x^4 + 9)$

Know it! Note

Methods to Factor Polynomials

Any Polynomial—Look for the greatest common factor.

$ab - ac = a(b - c)$	$6x^2y + 10xy^2 = 2xy(3x + 5y)$

Binomials—Look for a difference of two squares.

$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9y^2 = (x + 3y)(x - 3y)$

Trinomials—Look for perfect-square trinomials and other factorable trinomials.

$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 4x + 4 = (x + 2)^2$ $x^2 - 2x + 1 = (x - 1)^2$
$x^2 + bx + c = (x + \square)(x + \square)$ $ax^2 + bx + c = (\square x + \square)(\square x + \square)$	$x^2 + 3x + 2 = (x + 1)(x + 2)$ $6x^2 + 7x + 2 = (2x + 1)(3x + 2)$

Polynomials of Four or More Terms—Factor by grouping.

$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (x + y)(a + b)$	$2x^3 + 4x^2 + x + 2 = (2x^3 + 4x^2) + (x + 2)$ $= 2x^2(x + 2) + 1(x + 2)$ $= (x + 2)(2x^2 + 1)$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Give an expression that includes a polynomial that is not completely factored.
2. Give an example of an unfactorable binomial and an unfactorable trinomial.
3. **GET ORGANIZED** Copy the graphic organizer. Draw an arrow from each expression to the method you would use to factor it.

Know it! Note

Factoring Methods	
Polynomial	**Method**
1. $16x^4 - 25y^8$	A. Factoring out the GCF
2. $x^2 + 10x + 25$	B. Factoring by grouping
3. $9t^2 + 27t + 18t^4$	C. Unfactorable
4. $a^2 + 3a - 7a - 21$	D. Difference of two squares
5. $100b^2 + 81$	E. Perfect-square trinomial

3 Close

Summarize

Ask students to state the method they would use **first** to factor the following polynomials.

$16x^2 - 25$ difference of squares

$3x^2 + 12x + 27$ GCF

$4x^3 + 20x^2 + x + 5$ factor by grouping

$8x^2 - 32$ GCF

$x^2 + x - 12$ find factors for -12

Point out that no matter what the first step is, the resulting factorization is the same.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

Possible answers:

1. $x^2 - 1$
2. $x^2 + 1$; $x^2 + x + 1$
3. See Additional Answers.

7-6 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1

Tell whether each expression is completely factored. If not, factor it.

1. $3x(9x^2 + 1)$ yes
2. $2(4x^3 - 3x^2 - 8x)$
3. $2k^2(4 - k^3)$ yes
4. $(2x + 3)(3x - 5)$ yes
5. $4(4p^4 - 1)$
6. $a(a^3 + 2ab + b^2)$ yes

Factor each polynomial completely. Check your answer.

SEE EXAMPLE 2

7. $3x^5 - 12x^3$
8. $4x^3 + 8x^2 + 4x$
9. $8pq^2 + 8pq + 2p$
10. $18rs^2 - 2r$
11. $mn^5 - m^3n$
12. $2x^2y - 20xy + 50y$

SEE EXAMPLE 3

13. $6x^4 - 3x^3 - 9x^2$
14. $3y^2 + 14y + 4$
15. $p^5 + 3p^3 + p^2 + 3$
16. $7x^5 + 21x^4 - 28x^3$ $7x^3(x + 4)(x - 1)$
17. $2z^2 + 11z + 6$ unfactorable
18. $9p^2 - q^2 + 3p$ unfactorable

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–24	1
25–30	2
31–36	3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Tell whether each expression is completely factored. If not, factor it.

19. $2x(y^3 - 4y^2 + 5y)$
20. $2r(25r^6 - 36)$
21. $3n^2(n^2 - 25)$
22. $2m(m + 1)(m + 4)$ yes
23. $2y^2(4x^2 + 9)$ yes
24. $4(7g + 9h^2)$ yes

Factor each polynomial completely. Check your answer.

25. $-4x^3 + 24x^2 - 36x$
26. $24r^2 - 6r^4$
27. $5d^2 - 60d + 135$
28. $4y^8 + 36y^7 + 81y^6$
29. $98x^3 - 50xy^2$
30. $4x^3y - 4x^2y - 8xy$
31. $5x^2 - 10x + 14$
32. $121x^2 + 36y^2$
33. $p^4 - 16$
34. $4m^6 - 30m^5 + 36m^4$ $2m^4(m - 6)(2m - 3)$
35. $2k^3 + 3k^2 + 6k + 9$ $(k^2 + 3)(2k + 3)$
36. $ab^4 - 16a$ $a(b^2 + 4)(b + 2)(b - 2)$

Write an expression for each situation. Factor your expression.

37. the square of Ella's age plus 12 times Ella's age plus 36 $x^2 + 12x + 36 = (x + 6)^2$
38. the square of the distance from point A to point B minus 81 $d^2 - 81 = (d - 9)(d + 9)$
39. the square of the number of seconds Bob can hold his breath minus 16 times the number of seconds plus 28 $s^2 - 16s + 28 = (s - 2)(s - 14)$
40. three times the square of the number of apples on a tree minus 22 times the number of apples plus 35 $3a^2 - 22a + 35 = (a - 5)(3a - 7)$
41. the square of Beth's score minus 49 $b^2 - 49 = (b + 7)(b - 7)$
42. **Physics** The height in meters of a ballet dancer's center of mass when she leaps can be modeled by the polynomial $-5t^2 + 30t + 1$, where t is time in seconds after the jump. Tell whether the polynomial is fully factored when written as $-1(5t^2 - 30t - 1)$. Explain. Yes; $5t^2 - 30t - 1$ is unfactorable.
43. **Write About It** When asked to factor a polynomial completely, you first determine that the terms in the polynomial do not share any common factors. What would be your next step? Check for a pattern such as a perfect-square trinomial or a difference of 2 squares.

Factor and simplify each expression.

44. $12(x + 1)^2 + 60(x + 1) + 75$ $3(2x + 7)^2$
45. $(2x + 3)^2 - (x - 4)^2$ $(3x - 1)(x + 7)$
46. $45x(x - 2)^2 + 60x(x - 2) + 20x$ $5x(3x - 4)^2$
47. $(3x - 5)^2 - (y + 2)^2$ $(3x + y - 3)(3x - y - 7)$

Answers

2. no; $2x(4x^2 - 3x - 8)$
5. no; $4(2p^2 + 1)(2p^2 - 1)$
7. $3x^3(x + 2)(x - 2)$
8. $4x(x + 1)^2$
9. $2p(2q + 1)^2$
10. $2r(3s + 1)(3s - 1)$
11. $mn(n^2 + m)(n^2 - m)$
12. $2y(x - 5)^2$
13. $3x^2(2x - 3)(x + 1)$
14. unfactorable
15. $(p^3 + 1)(p^2 + 3)$
19. no; $2xy(y^2 - 4y + 5)$
20. no; $2r(5r^3 + 6)(5r^3 - 6)$
21. no; $3n^2(n + 5)(n - 5)$
25. $-4x(x - 3)^2$
26. $-1(6r^2)(r - 2)(r + 2)$
27. $5(d - 3)(d - 9)$
28. $y^6(2y + 9)^2$
29. $2x(7x + 5y)(7x - 5y)$
30. $4xy(x - 2)(x + 1)$
31. unfactorable
32. unfactorable
33. $(p^2 + 4)(p + 2)(p - 2)$

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

Basic 19–48, 50–58
Average 19–42, 44–58
Advanced 19–36, 40, 42, 44–64

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 22, 28, 32, 36, 42

Teaching Tip

Inclusion For **Exercises 25–36,** have students count the number of terms in the polynomial when deciding which factoring method to use. Four or more terms could indicate factoring by grouping, three terms could indicate the pattern for a perfect-square trinomial, and two terms could indicate the pattern for the difference of two squares.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 42, 48, 59, 64

Construct viable arguments and critique the reasoning of others. Exercises 43, 49, 51

Attend to precision. Exercise 50

Look for and express regularity in repeated reasoning. Exercises 52–54

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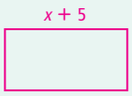
MULTI-STEP TEST PREP **Exercise 48** involves factoring a polynomial to find the dimensions of a rectangle. This exercise prepares students for the Multi-Step Test Prep.

TEST PREP DOCTOR In **Exercise 56,** students who chose **F** may have factored out the GCF but forgotten to factor by using the difference of squares. Students who chose **G** may have factored by using the difference of squares but forgotten to factor out the GCF.

Students who chose **B** in **Exercise 57** may have forgotten to keep the subtraction sign with the 1 when using the Distributive Property.

Answers

48b.



49. (1) $4x^2 - 100 = 4(x^2 - 25) = 4(x-5)(x+5)$

(2) $4x^2 - 100 = (2x+10)(2x-10) = 2(x+5)(2)(x-5) = 4(x+5)(x-5)$

MULTI-STEP TEST PREP

48. **a.** The area of a Marci's rectangular flower garden is $(x^2 + 2x - 15)$ ft^2. Factor this expression. $(x+5)(x-3)$

b. Draw a diagram of the garden and label the length and width with your factors from part **a.**

c. Find the length and width of the flower garden if $x = 7$ ft. $\ell = 12$ ft; $w = 4$ ft

49. **Critical Thinking** Show two methods of factoring $4x^2 - 100$.

50. **Estimation** Estimate the value of $2x^2 + 5xy + 3y^2$ when $x = -10.1$ and $y = 10.05$. (*Hint:* Factor the expression first.) approx. 0

51. **ERROR ANALYSIS** Examine the factorization shown. Explain why the factorization is incorrect. Possible answer: $(2x-1)^2 \neq 4x^2 - 4x - 1$

$12x^2 - 12x - 3$
$3(4x^2 - 4x - 1)$
$3(2x - 1)(2x - 1)$

LINK Math History

Blaise Pascal was a French mathematician who lived in the 1600s.

Math History **Use the following information for Exercises 52–54.**

The triangle at right is called *Pascal's Triangle*. The triangle starts with 1 and each of the other numbers in the triangle is the sum of the two numbers in the row above it.

0					1					
1				1		1				
2			1		2		1			
3		1		3		3		1		
4	1		4		6		4		1	
5	1	5		10		10		5		1

Pascal's Triangle can be used to write the product of a binomial raised to an integer power. The numbers in each row give you the coefficients of each term in the product.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

The numbers in row **3** are 1, 3, 3, 1. These are the coefficients of the terms in the product $(a+b)^3$. The power of a decreases in each term and the power of b increases in each term.

Use the patterns you see in Pascal's Triangle to write the power of the binomial $a + b$ given by each polynomial.

52. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 = (a+b)^6$

53. $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8 = (a+b)^8$

54. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7 = (a+b)^7$

TEST PREP

55. Which expression equals $6x^2 + 7x - 10$?

Ⓐ $(6x+2)(x-5)$ Ⓒ $(x+2)(6x-5)$
Ⓑ $(2x+5)(3x-2)$ Ⓓ $(3x+2)(2x-5)$

56. What is the complete factorization of $16x^{12} - 256$?

Ⓕ $16(x^6+4)(x^6-4)$ Ⓗ $16(x^6+4)(x^3+2)(x^3-2)$
Ⓖ $(4x^6+16)(4x^6-16)$ Ⓙ $(4x^6+16)(2x^3+4)(2x^3-4)$

(tl), © Stockbyte; (cb), Sheila Terry/Science Photo Library/Photo Researchers, Inc.

PRACTICE A

PRACTICE C

PRACTICE B

Tell whether each polynomial is completely factored. If not, factor it.

1. $6(t^2+12)$ — yes
2. $5(m^2+9m)$ — no; $5m(m+9)$
3. $2p(p^4-9)$ — no; $2p(p^2+3)(p^2-3)$
4. $(x-8)(2x+3)$ — yes
5. $3k^2(5k^2+19)$ — yes
6. $7(14g^4-4g+10)$ — no; $14(7g^4-2g+5)$

Factor each polynomial completely.

7. $24x+40$ — $8(3x+5)$
8. $5r^2-10r$ — $5r(r^2-2)$
9. $3x^2y+x^2y^2$ — $x^2y(3x+y)$
10. $-3a^2b+12ab-12b$ — $-3b(a-2)^2$
11. $5t^3-45t+3t^2-27$ — $(5t+3)(t+3)(t-3)$
12. $2y^2-6y-56$ — $2(y+4)(y-7)$
13. $6a^3+39a^2+45a$ — $3a(2a+3)(a+5)$
14. x^3-9x — $x(x-3)(x+3)$
15. $12n^3-48$ — $12(n^3-4)$
16. $3c^4+24c^2+48c^2$ — $3c^2(c+4)^2$
17. $3d^6+4d-2$ — unfactorable
18. $10w^6-160w^2$ — $10w^2(w^2+4)(w-2)(w+2)$

READING STRATEGIES

Use the sequence chain below to guide you in factoring polynomials.

Factoring Polynomials

Is there a GCF among the terms? If yes, factor it out. If no, move on.

How many terms are in the polynomial? If 2 / If 3 / If 4

Is the binomial a difference of squares $a^2 - b^2$? If yes, factor it into $(a+b)(a-b)$. If no, you're done.

Is this a perfect square trinomial? If yes, factor it into $(a+b)^2$ or $(a-b)^2$. If no,

Are there terms with common factors? If yes, try factor by grouping. If no, you're done.

Can this trinomial be factored into 2 binomials? If yes, factor it. If no, you're done.

Complete the following.

1. What is the first thing to look for when factoring any polynomial? GCF
2. If the polynomial has 2 terms, what special product should you look for? difference of squares
3. Describe the first step in factoring the polynomial $16x^2 + 12x - 20$. Factor 4 out

Factor each polynomial using the sequence chain as a guide.

4. $2x^2 - 15x - 27$ — $(2x+3)(x-9)$
5. $3x^2 - 75$ — $3(x+5)(x-5)$
6. $18x^4 + 12x^2 + 2$ — $2(3x^2+1)^2$

RETEACH

Use the following table to help you choose a factoring method.

First factor out a GCF if possible. Then,

If binomial,	check for difference of squares.	yes / no	Use $(a+b)(a-b)$. If no GCF, it cannot be factored.
If trinomial,	check for perfect square trinomial.	yes / no	Factor using $(a+b)^2$ or $(a-b)^2$. If $a = 1$, check factors of c that sum to b. If $a \neq 1$, check inner plus outer factors of a and c that sum to b.
If 4 or more terms,			Try to factor by grouping.

Explain how to choose a factoring method for $x^2 - x - 30$. Then state the method.

- There is no GCF.
- $x^2 - x - 30$ is a trinomial.
- The terms a and b are not perfect squares, therefore this is not a perfect square trinomial.
- $a = 1$

Method: Factor by checking factors of c that sum to b.

Explain how to choose a factoring method for $2x^2 - 50$. Then state the method.

- Factor out the GCF: $2(x^2 - 25)$
- $x^2 - 25$ is a binomial.
- a and b are perfect squares. This is a difference of squares.

Method: Factor out GCF. Then use $(a+b)(a-b)$.

Explain how to choose a factoring method for each polynomial. Then state the method.

1. $x^2 + 14x + 49$ — no GCF; $x^2 + 14x + 49$ is a trinomial; This is a perfect square trinomial. Method: use $(a+b)^2$.
2. $4x^2 - 40$ — factor out the GCF: $4(x^2 - 10)$; $x^2 - 10$ is a binomial; This is not a difference of squares. Method: Factor out GCF.
3. $2x^2 + 8x + 6$ — factor out the GCF: $2(x^2 + 4x + 3)$; $x^2 + 4x + 3$ is a trinomial; This is not a perfect square trinomial; $a = 1$; Method: Factor out GCF. Then find factors of c that sum to b.

57. Which of the expressions below represents the fifth step of the factorization?

Step 1: $40a^3 - 60a^2 - 10a + 15$

Step 2: $5(8a^3 - 12a^2 - 2a + 3)$

Step 3: $5[(8a^3 - 12a^2) - (2a - 3)]$

Step 4: $5[4a^2(2a - 3) - 1(2a - 3)]$

Step 5: _____

Step 6: $5(2a - 3)(2a + 1)(2a - 1)$

(A) $5(2a - 3)(2a + 3)(4a^2 - 1)$
(B) $5(2a - 3)(4a^2 + 1)$
(C) $5(2a - 3)(4a^2 - 1)$
(D) $5(2a - 3)(2a - 3)(4a^2 - 1)$

58. Short Response Use the polynomial $8x^3 + 24x^2 + 18x$ for the following.

a. Factor the polynomial. Explain each step and tell whether you used any rules for special products.

b. Explain another set of steps that could be used to factor the polynomial.

The polynomial could be factored by finding factors of 8 and factors of 18 that would result in 24 as the sum of the outer and inner products. Then one binomial would need to be factored again.

CHALLENGE AND EXTEND

59. Geometry The volume of the cylinder shown is represented by the expression $72\pi p^3 + 48\pi p^2 + 8\pi p$. The height of the cylinder is $8p$.

a. Factor the expression for volume. $V = 8p[\pi(3p + 1)^2]$

b. What expression represents the radius of the cylinder? $r = (3p + 1)$ cm

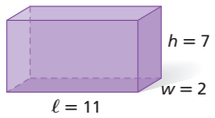
c. If the radius is 4 cm, what are the height and volume of the cylinder? $h = 8$ cm; $V = 128\pi$ cm^3

Factor.

60. $g^7 + g^3 + g^5 + g^4$ $g^3(g^4 + g^2 + g + 1)$ **61.** $h^2 + h^8 + h^6 + h^4$ $h^2(h^4 + 1)(h^2 + 1)$

62. $x^{n+2} + x^{n+1} + x^n$ $x^n(x^2 + x + 1)$ **63.** $x^{n+5} + x^{n+4} + x^{n+3}$ $x^{n+3}(x^2 + x + 1)$

64. Geometry The rectangular prism has the dimensions shown.



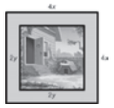
a. Write expressions for the height and length of the prism using w. $h = w + 5$; $\ell = w + 9$

b. Write a polynomial that represents the volume of the prism using w. $V = w^3 + 14w^2 + 45w$

PROBLEM SOLVING

Write the correct answer.

1. A rectangular stage set up in a theater has an area of $(15x^2 + 3x - 12)$ square feet. Factor the polynomial completely.
   $3(5x - 4)(x + 1)$
3. An artist framed a picture. The dimensions of the picture and frame are shown below.



Completely factor the expression for the area of the frame.
$4(2x + y)(2x - y)$

2. The area of a circular rug is $(16\pi k^2 - 16\pi k + 4\pi)$ m^2. Factor the expression completely. Then find the area of the rug if $k = 1$ meter.
   $4\pi(2k - 1)^2$; 4π m^2
4. The attendance for a team's basketball game can be approximated with the polynomial $-5x^2 + 80x + 285$, where x is the number of wins the team had in the previous month. Factor the polynomial completely. Then estimate the attendance when the team won 4 games in the previous month.
   $-5(x - 19)(x + 3)$; 525 attendees

Select the best answer.

5. The volume of a box can be modeled by the expression $7x^4 - 28$. Which shows this expression completely factored?
   A $7(x^4 - 4)$
   B $7(x^2 - 2)^2$
   C $(7x^2 + 4)(x^2 - 7)$
   D $7(x^2 + 2)(x^2 - 2)$
6. The area of a Japanese rock garden is $(30x^2 + 3x - 6)$ square feet. Factor the polynomial completely.
   F $3(10x^2 + x - 2)$
   G $3(2x + 1)(5x - 2)$
   H $(6x + 3)(5x - 2)$
   J $(15x - 6)(2x + 1)$
7. The money made from the sales of x mountain bikes is approximated by $20x^2 + 10x + 90$. Factor the expression completely.
   A $2(10x + 9)(x + 5)$
   B $5(4x^2 + 2x + 18)$
   C $10(2x^2 + x + 9)$
   D The expression cannot be factored.
8. Kyle stood on a bridge and threw a rock up and over the side. The height of the rock, in meters, can be approximated by $-5t^2 + 5t + 24$, where t is the time in seconds after Kyle threw it. Completely factor the expression.
   F $-5(t^2 + t + 24)$
   G $(-5t + 3)(t + 8)$
   H $-1(5t + 8)(t + 3)$
   J The expression cannot be factored.

CHALLENGE

A sum of cubes can be factored using: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Factor $x^3 + 125y^3$.

$(x)^3 + (5y)^3$ — $a = x$, $b = 5y$

$(x + 5y)(x^2 - 5xy + 25y^2)$ — *Substitute into* $(a + b)(a^2 - ab + b^2)$.

Factor each polynomial.

1. $x^3 + 8$ $(x + 2)(x^2 - 2x + 4)$
2. $64x^3 + y^3$ $(4x + y)(16x^2 - 4xy + y^2)$
3. $125x^3 + 216$ $(5x + 6)(25x^2 - 30x + 36)$

A difference of cubes can be factored using: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Factor $8x^3 - 27$.

$(2x)^3 - (3)^3$ — $a = 2x$, $b = 3$

$(2x - 3)(4x^2 + 6x + 9)$ — *Substitute into* $(a - b)(a^2 + ab + b^2)$.

Factor each polynomial.

4. $x^3 - 64$ $(x - 4)(x^2 + 4x + 16)$
5. $343x^3 - y^3$ $(7x - y)(49x^2 + 7xy + y^2)$
6. $729x^3 - 1000$ $(9x - 10)(81x^2 + 90x + 100)$

You can use the sum and difference of cubes with other factoring methods to completely factor a polynomial.

Factor each polynomial completely.

7. $2x^3 - 54$ $2(x - 3)(x^2 + 3x + 9)$
8. $216x^3 + 8y^3$ $8(3x + y)(9x^2 - 3xy + y^2)$
9. $x^5 + 12x^3 + 32$ $(x + 2)(x^2 + 2x + 4)(x^3 + 4)$
10. $3x^8 + 18x^3 + 15$ $3(x^3 + 5)(x + 1)(x^2 - x + 1)$
11. $x^5 - 27x^2 - 9x^3 + 243$ $(x + 3)(x - 3)^2(x^2 + 3x + 9)$

Use difference of squares to factor each binomial. Then factor completely.

12. $729x^6 - 64$ $(3x + 2)(3x - 2)(9x^2 - 6x + 4)(9x^2 + 6x + 4)$
13. $64x^6 - y^6$ $(2x + y)(2x - y)(4x^2 - 2xy + y^2)(4x^2 + 2xy + y^2)$

Teaching Tip **Geometry** In **Exercise 59,** students must factor an expression that represents the volume of a cylinder. Remind students that π is a number and can be factored out just like any other number.

Answers

58a. $8x^3 + 24x^2 + 18x = 2x(4x^2 + 12x + 9) = 2x(2x + 3)^2$; first I factored out the GCF, $2x$, and then I used the pattern for a perfect-square trinomial.

Journal

Explain what you think is the easiest type of polynomial to factor. Use an example to demonstrate.

ALTERNATIVE ASSESSMENT

Pair students. Have each student work backward to create three different polynomials that require at least two different factoring methods. Then have students switch papers and factor their partner's polynomials.

Power Presentations with PowerPoint®

7-6 Lesson Quiz

Tell whether each expression is completely factored. If not, factor it.

1. $(x + 3)(5x + 10)$
   no; $5(x + 3)(x + 2)$
2. $3x^2(x^2 + 9)$
   yes

Factor each polynomial completely. Check your answer.

3. $x^3 + 4x^2 + 3x + 12$
   $(x + 4)(x^2 + 3)$
4. $4x^2 + 16x - 48$
   $4(x + 6)(x - 2)$
5. $18x^2 - 3x - 3$
   $3(3x + 1)(2x - 1)$
6. $18x^2 - 50y^2$
   $2(3x + 5y)(3x - 5y)$
7. $5x - 20x^3 + 7 - 28x^2$
   $(1 + 2x)(1 - 2x)(5x + 7)$

Also available online

SECTION 7B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

Answers

1. $\ell = (x + 6)$ ft; $w = (x + 6)$ ft
2. $w = 12$ ft; $x = 6$; $A = 144$ ft^2
3. $\ell = (x + 12)$ ft; $w = (x + 2)$ ft

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them

Learn It Online
State Resources Online

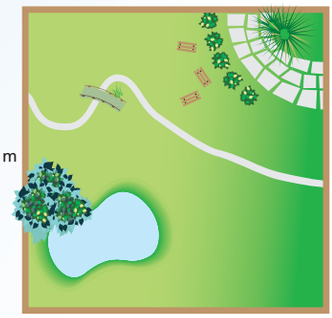
MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Factoring

Shaping the Environment The Environmental Awareness Club is going to plant a garden on the front lawn of the school. Henry suggests a garden in the shape of a square. Theona suggests a rectangular shape.

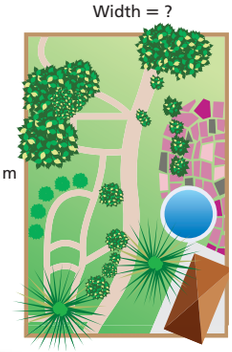
Width = ?



Length = 12 m

1. Henry's plans include a square garden with an area of $(x^2 + 12x + 36)$ m^2. Write expressions for the length and width of the square garden.
2. A drawing of the square garden shows a length of 12 m. What is the width of the square garden? What is the value of x? What is the total area of the square garden?
3. Theona's plans include a rectangular garden with an area of $(x^2 + 14x + 24)$ m^2. Write expressions for the length and width of the rectangular garden.
4. A drawing of the rectangular garden shows that the length is 6 m longer than the length of the square garden. What is the width of the rectangular garden? How much shorter is the width of the rectangular garden than the square garden? **$w = 8$ m; 4 m**
5. Find the perimeter of each garden in terms of x.
6. Which plan should the club choose if they want the garden that covers the most area? Which plan should the club choose if they want the garden that requires the least fencing around it? Explain your reasoning.

Width = ?



Length = (12 + 6) m

5. square garden: $4(x + 6)$ m; rectangular garden: $4(x + 7)$ m

6. Area of square garden: 144 m^2; area of rectangular garden: 144 m^2; the gardens have the same area; the perimeter of the square garden is less and requires less fencing.

(tl), ©Stockbyte; (br), David Young Wolff/Photoedit

INTERVENTION

Scaffolding Questions

1. When given the area, how can you find the length and width? Factor the area.
2. What values of x are reasonable? x must be positive.
3. Will the length and width be equal if the rectangle is not a square? no
4. Does it matter which side of the square garden you label as the width? No; the sides are all the same length.
5. What does perimeter describe? the distance around a figure
6. How do you know which garden requires the least fencing? The garden with the smaller perimeter requires less fencing.

Extension

Do you think the perimeter of two gardens with the same area will be larger for a square or rectangle? The perimeter will be larger for the rectangle.

Quiz for Lessons 7-5 Through 7-6

7-5 Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 + 8x + 16$ yes; $(x + 4)^2$
2. $4x^2 - 20x + 25$ yes; $(2x - 5)^2$
3. $x^2 + 3x + 9$ no; $3x \neq 2(x)(3)$
4. $2x^2 - 4x + 4$
5. $9x^2 - 12x + 4$
6. $x^2 - 12x - 36$
7. An architect is designing square windows with an area of $(x^2 + 20x + 100)\ \text{ft}^2$. The dimensions of the windows are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the windows. Find the perimeter of a window when $x = 4$ ft. $4(x + 10)$ ft; 56 ft

Determine whether each trinomial is a difference of two squares. If so, factor. If not, explain.

8. $x^2 - 121$
9. $4t^2 - 20$
10. $1 - 9y^4$
11. $25m^2 - 4m^6$
12. $16x^2 + 49$
13. $r^4 - t^2$
14. The area of a square is $(36d^2 - 36d + 9)\ \text{in}^2$.
    a. What expression represents the length of a side of the square? $\ell = (6d - 3)$ in.
    b. What expression represents the perimeter of the square? $4(6d - 3)$ in.
    c. What are the length of a side, the perimeter, and the area of the square when $d = 2$ in.? 9 in.; 36 in.; 81 in^2

7-6 Choosing a Factoring Method

Tell whether each expression is completely factored. If not, factor it.

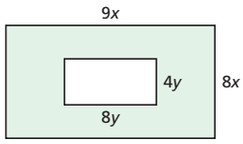
15. $5(x^2 + 3x + 1)$ yes
16. $6x(5x^2 - x)$ no; $6x^2(5x - 1)$
17. $3t(t^4 - 9)$ no; $3t(t^2 - 3)(t^2 + 3)$
18. $2(m^2 - 10m + 25)$ no; $2(m - 5)^2$
19. $3(2y^2 - 5)(y + 1)$ yes
20. $(2n + 6)(n - 4)$ no; $2(n + 3)(n - 4)$

Factor each polynomial completely. Check your answer.

21. $3x^3 - 12x^2 + 12x$
22. $16m^3 - 4m$
23. $5x^3y - 45xy$
24. $3t^2 + 5t - 1$
25. $3c^2 + 12c - 63$
26. $x^5 - 81x$

Write an expression for each situation. Then factor your expression.

27. the difference of the square of a board's length and 36
28. the square of Michael's age minus 8 times Michael's age plus 16
29. two times the square of a car's speed plus 2 times the car's speed minus 12
30. three times the cube of Jessie's height plus 3 times the square of Jessie's height minus 6 times Jessie's height
31. Write an expression for the area of the shaded region. Then factor the expression.



READY TO GO ON? SECTION 7B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources

Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

4–6, 8–13, 21–31. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 7B		
	Worksheets	Online	
Lesson 7-5	7-5 Intervention	Activity 7-5	Diagnose and Prescribe Online
Lesson 7-6	7-6 Intervention	Activity 7-6	

***Ready to Go On? Enrichment*, Section 7B**

Worksheets

Online

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Lesson Tutorial Videos

Answers

1. prime factorization
2. greatest common factor
3. $2^2 \cdot 3$
4. $2^2 \cdot 5$
5. 2^5
6. prime
7. $2^3 \cdot 5$
8. 2^6
9. $2 \cdot 3 \cdot 11$
10. $2 \cdot 3 \cdot 19$
11. 5
12. 12
13. 1
14. 27
15. 4
16. 3
17. $2x$
18. $9b^2$
19. $25r$
20. 6 boxes; 13 rows

CHAPTER 7 Study Guide: Review

Vocabulary

greatest common factor **prime factorization**

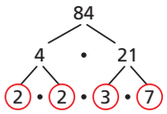
Complete the sentences below with vocabulary words from the list above.

1. A number written as a product so that each of its factors has no factors other than 1 and itself is the ___?___.
2. The ___?___ of two monomials is the greatest of the factors that the monomials share.

7-1 Factors and Greatest Common Factors

EXAMPLES

■ **Write the prime factorization of 84.**



Write as a product. Continue until all factors are prime.

■ **Write the prime factorization of 75.**

Keep dividing by prime factors until the quotient is 1.

$75 = 3 \cdot 5 \cdot 5 = 3 \cdot 5^2$

■ **Find the GCF of 36 and 90.**

$36 = 2 \cdot 2 \cdot 3 \cdot 3$
$90 = 2 \cdot 3 \cdot 3 \cdot 5$
$2 \cdot 3 \cdot 3 = 18$

Write the prime factorization of each number.
Find the product of the common factors.

The GCF of 36 and 90 is 18.

■ **Find the GCF of $10x^5$ and $4x^2$.**

$10x^5 = 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x$
$4x^2 = 2 \cdot 2 \cdot x \cdot x$
$2 \cdot x \cdot x = 2x^2$

Write the prime factorization of each coefficient.
Write powers as products.
Find the product of the common factors.

The GCF of $10x^5$ and $4x^2$ is $2x^2$.

EXERCISES

Write the prime factorization of each number.

3. 12
4. 20
5. 32
6. 23
7. 40
8. 64
9. 66
10. 114

Find the GCF of each pair of numbers.

11. 15 and 50
12. 36 and 132
13. 29 and 30
14. 54 and 81
15. 20 and 48

Find the GCF of each pair of monomials.

16. $9m$ and 3
17. $4x$ and $2x^2$
18. $-18b^4$ and $27b^2$
19. $100r$ and $25r^5$
20. A hardware store carries 42 types of boxed nails and 36 types of boxed screws. The store manager wants to build a rack so that he can display the hardware in rows. He wants to put the same number of boxes in each row, but he wants no row to contain both nails and screws. What is the greatest number of boxes that he can display in one row? How many rows will there be if the manager puts the greatest number of boxes in each row?

7-2 Factoring by GCF

EXAMPLES

■ **Factor $3t^3 - 9t^2$. Check your answer.**

$3t^3 = 3 \cdot t \cdot t \cdot t$
$9t^2 = 3 \cdot 3 \cdot t \cdot t$ *Find the GCF.*

GCF: $3 \cdot t \cdot t = 3t^2$

$3t^3 - 9t^2 = 3t^2(t) - 3t^2(3)$
$= 3t^2(t - 3)$ *Factor out the GCF.*

Check $3t^2(t-3) = 3t^3 - 9t^2$ ✓

■ **Factor $-12s - 6s^3$. Check your answer.**

$-1(12s + 6s^3)$ *Factor out -1.*

$12s = 2 \cdot 2 \cdot 3 \cdot s$
$6s^3 = 2 \cdot \quad 3 \cdot s \cdot s \cdot s$ *Find the GCF.*

GCF: $2 \cdot 3 \cdot s = 6s$

$-1(12s + 6s^3)$
$-1[(6s)(2) + (6s)(s^2)]$
$-1[(6s)(2 + s^2)]$
$-6s(2 + s^2)$ *Factor out the GCF.*

Check $-6s(2 + s^2) = -12s - 6s^3$ ✓

■ **Factor $5(x - 7) + 3x(x - 7)$ by grouping.**

$5(x-7) + 3x(x-7)$ *$(x - 7)$ is a common factor.*

$(x-7)(5 + 3x)$ *Factor out $(x - 7)$.*

■ **Factor $6b^3 + 8b + 15b^2 + 20$ by grouping.**

$(6b^3 + 8b) + (15b^2 + 20)$ *Group terms that have a common factor.*

$2b(3b^2 + 4) + 5(3b^2 + 4)$ *Factor each group.*

$(3b^2 + 4)(2b + 5)$ *Factor out $(3b^2 + 4)$.*

■ **Factor $2m^3 - 6m^2 + 15 - 5m$. Check your answer.**

$(2m^3 - 6m^2) + (15 - 5m)$ *Group terms.*
$2m^2(m - 3) + 5(3 - m)$ *Factor each group.*

$2m^2(m - 3) + 5(-1)(m - 3)$ *Rewrite $(3 - m)$ as $(-1)(m - 3)$.*

$2m^2(m - 3) - 5(m - 3)$ *Simplify.*

$(m - 3)(2m^2 - 5)$ *Factor out $(m - 3)$.*

Check $(m - 3)(2m^2 - 5)$

$2m^3 - 5m - 6m^2 + 15$
$2m^3 - 6m^2 + 15 - 5m$ ✓

EXERCISES

Factor each polynomial. Check your answer.

21. $5x - 15x^3$
22. $-16b + 32$
23. $-14v - 21$
24. $4a^2 - 12a - 8$
25. $5g^5 - 10g^3 - 15g$
26. $40p^2 - 10p + 30$

27. A civil engineer needs the area of a rectangular lot to be $(6x^2 + 5x)$ ft^2. Factor this polynomial to find possible expressions for the dimensions of the lot.

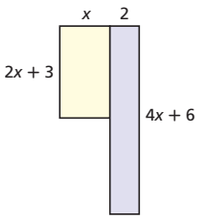
Factor each expression.

28. $2x(x - 4) + 9(x - 4)$
29. $t(3t + 5) - 6(3t + 5)$
30. $5(6 - n) - 3n(6 - n)$
31. $b(b + 4) + 2(b + 4)$
32. $x^2(x - 3) + 7(x - 3)$

Factor each polynomial. Check your answer.

33. $n^3 + n - 4n^2 - 4$
34. $6b^2 - 8b + 15b - 20$
35. $2h^3 - 7h + 14h^2 - 49$
36. $3t^2 + 18t + t + 6$
37. $10m^3 + 15m^2 - 2m - 3$
38. $8p^3 + 4p - 6p^2 - 3$
39. $5r - 10 + 2r - r^2$
40. $b^3 - 5b + 15 - 3b^2$
41. $6t - t^3 - 4t^2 + 24$
42. $12h - 3h^2 + h - 4$
43. $d - d^2 + d - 1$
44. $6b - 5b^2 + 10b - 12$
45. $5t - t^2 - t + 5$
46. $8b^2 - 2b^3 - 5b + 20$
47. $3r - 3r^2 - 1 + r$

48. Write an expression for the area of each of the two rectangles shown. Then write and factor an expression for the combined area.



Answers

21. $5x(1 - 3x^2)$
22. $16(-b + 2)$
23. $-7(2v + 3)$
24. $4(a^2 - 3a - 2)$
25. $5g(g^2 - 3)(g^2 + 1)$
26. $10(4p^2 - p + 3)$
27. $(6x + 5)$ ft by x ft
28. $(2x + 9)(x - 4)$
29. $(t - 6)(3t + 5)$
30. $(5 - 3n)(6 - n)$
31. $(b + 2)(b + 4)$
32. $(x^2 + 7)(x - 3)$
33. $(n^2 + 1)(n - 4)$
34. $(2b + 5)(3b - 4)$
35. $(2h^2 - 7)(h + 7)$
36. $(3t + 1)(t + 6)$
37. $(5m^2 - 1)(2m + 3)$
38. $(4p - 3)(2p^2 + 1)$
39. $-1(r - 5)(r - 2)$
40. $(b^2 - 5)(b - 3)$
41. $(t + 4)(-t^2 + 6)$
42. $-1(3h - 1)(h - 4)$
43. $-1(d - 1)^2$
44. $(2 - b)(5b - 6)$
45. $(t + 1)(5 - t)$
46. $(2b^2 + 5)(4 - b)$
47. $-1(3r - 1)(r - 1)$
48. left rectangle: $2x^2 + 3x$; right rectangle: $8x + 12$; combined: $2x^2 + 8x + 3x + 12$; $(2x + 3)(x + 4)$

Answers

49. $(x + 1)(x + 5)$
50. $(x + 2)(x + 4)$
51. $(x + 3)(x + 5)$
52. $(x - 6)(x - 2)$
53. $(x + 5)^2$
54. $(x - 2)(x - 11)$
55. $(x + 4)(x + 20)$
56. $(x - 6)(x - 20)$
57. $(x + 12)(x - 7)$
58. $(x + 3)(x - 8)$
59. $(x + 4)(x - 7)$
60. $(x - 1)(x + 5)$
61. $(x + 3)(x - 2)$
62. $(x + 5)(x - 4)$
63. $(x - 8)(x + 6)$
64. $(x - 9)(x + 4)$
65. $(x - 12)(x + 6)$
66. $(x - 10)(x + 7)$
67. $(x + 20)(x - 6)$
68. $(x + 7)(x - 1)$
69. $(y + 3)$ m
70. $(2x + 1)(x + 5)$
71. $(3x + 7)(x + 1)$
72. $(2x - 1)(x - 1)$
73. $(3x + 2)(x + 2)$
74. $(5x + 3)(x + 5)$
75. $(2x - 3)(3x - 5)$
76. $(4x + 5)(x + 2)$
77. $(3x + 4)(x + 2)$
78. $(7x - 2)(x - 5)$
79. $(3x + 2)(3x + 4)$
80. $(2x + 1)(x - 1)$
81. $(3x + 1)(x - 4)$
82. $(2x - 1)(x - 5)$
83. $(7x + 2)(x - 3)$
84. $(5x + 1)(x - 2)$
85. $-1(2x - 1)(3x + 2)$
86. $(6x + 5)(x - 1)$
87. $(3x - 2)(2x + 7)$
88. $-1(2x + 1)(2x - 5)$
89. $-1(2x - 3)(5x + 2)$
90. $12x^2 - 11x - 5$; $(4x - 5)(3x + 1)$

7-3 Factoring $x^2 + bx + c$

EXAMPLES

Factor each trinomial. Check your answer.

■ $x^2 + 14x + 45$

$(x + \square)(x + \square)$ *Look for factors of 45 whose sum is 14.*

$(x + 9)(x + 5)$

Check $(x + 9)(x + 5) = x^2 + 5x + 9x + 45$

$= x^2 + 14x + 45$ ✓

■ $x^2 + 6x - 27$

$(x + \square)(x - \square)$ *Look for factors of −27 whose sum is 6.*

$(x + 9)(x - 3)$

Check $(x + 9)(x - 3) = x^2 - 3x + 9x - 27$

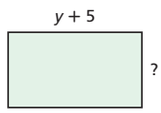
$= x^2 + 6x - 27$ ✓

EXERCISES

Factor each trinomial. Check your answer.

49. $x^2 + 6x + 5$
50. $x^2 + 6x + 8$
51. $x^2 + 8x + 15$
52. $x^2 - 8x + 12$
53. $x^2 + 10x + 25$
54. $x^2 - 13x + 22$
55. $x^2 + 24x + 80$
56. $x^2 - 26x + 120$
57. $x^2 + 5x - 84$
58. $x^2 - 5x - 24$
59. $x^2 - 3x - 28$
60. $x^2 + 4x - 5$
61. $x^2 + x - 6$
62. $x^2 + x - 20$
63. $x^2 - 2x - 48$
64. $x^2 - 5x - 36$
65. $x^2 - 6x - 72$
66. $x^2 - 3x - 70$
67. $x^2 + 14x - 120$
68. $x^2 + 6x - 7$

69. The rectangle shown has an area of $(y^2 + 8y + 15)$ m^2. What is the width of the rectangle?



7-4 Factoring $ax^2 + bx + c$

EXAMPLES

Factor each trinomial.

■ $6x^2 + 17x + 5$

$(\square x + \square)(\square x + \square)$ *$a = 6$ and $c = 5$; Outer + Inner = 17*

Factors of 6	Factors of 5	Outer + Inner	
1 and 6	5 and 1	$1(1) + 6(5) = 31$	✗
2 and 3	1 and 5	$2(5) + 3(1) = 13$	✗
2 and 3	5 and 1	$1(1) + 3(5) = 17$	✓

$(2x + 5)(3x + 1)$

■ $2n^2 - n - 10$

$(\square n + \square)(\square n + \square)$ *$a = 2$ and $c = -10$; Outer + Inner = −1*

Factors of 2	Factors of −10	Outer + Inner	
1 and 2	1 and −10	$1(-10) + 2(1) = -8$	✗
1 and 2	−1 and 10	$1(10) + 2(-1) = 8$	✗
1 and 2	2 and −5	$1(-5) + 2(2) = -1$	✓

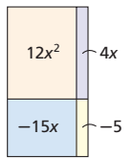
$(1n + 2)(2n - 5) = (n + 2)(2n - 5)$

EXERCISES

Factor each trinomial. Check your answer.

70. $2x^2 + 11x + 5$
71. $3x^2 + 10x + 7$
72. $2x^2 - 3x + 1$
73. $3x^2 + 8x + 4$
74. $5x^2 + 28x + 15$
75. $6x^2 - 19x + 15$
76. $4x^2 + 13x + 10$
77. $3x^2 + 10x + 8$
78. $7x^2 - 37x + 10$
79. $9x^2 + 18x + 8$
80. $2x^2 - x - 1$
81. $3x^2 - 11x - 4$
82. $2x^2 - 11x + 5$
83. $7x^2 - 19x - 6$
84. $5x^2 - 9x - 2$
85. $-6x^2 - x + 2$
86. $6x^2 - x - 5$
87. $6x^2 + 17x - 14$
88. $-4x^2 + 8x + 5$
89. $-10x^2 + 11x + 6$

90. Write the polynomial modeled and then factor.



7-5 Factoring Special Products

EXAMPLES

■ **Determine whether $x^2 + 18x + 81$ is a perfect square. If so, factor. If not, explain.**

$x^2 + 18x + 81$ — *The trinomial is of the form $a^2 + 2ab + b^2$, so it is a perfect-square trinomial.*

$x \cdot x \quad 2(x \cdot 9) \quad 9 \cdot 9$

$x^2 + 18x + 81 = (x + 9)^2$

■ **Determine whether $49x^4 - 25y^6$ is a difference of two squares. If so, factor. If not, explain.**

$49x^4 - 25y^6$ — *The binomial is a difference of two squares.*

$7x^2 \cdot 7x^2 \quad 5y^3 \cdot 5y^3$

$(7x^2)^2 - (5y^3)^2$ — *$a = 7x^2$, $b = 5x^3$*

$(7x^2 + 5y^3)(7x^2 - 5y^3)$ — *Write the binomial as $(a + b)(a - b)$.*

$49x^4 - 25y^6 = (7x^2 + 5y^3)(7x^2 - 5y^3)$

EXERCISES

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

91. $x^2 + 12x + 36$ **92.** $x^2 + 5x + 25$

93. $4x^2 - 2x + 1$ **94.** $9x^2 + 12x + 4$

95. $16x^2 + 8x + 4$ **96.** $x^2 + 14x + 49$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

97. $100x^2 - 81$ **98.** $x^2 - 2$

99. $5x^4 - 10y^6$ **100.** $(-12)^2 - (x^3)^2$

101. $121b^2 + 9c^8$ **102.** $100p^2 - 25q^2$

Factor each polynomial using the rule for perfect-square trinomials or the rule for a difference of two squares. Tell which rule you used.

103. $x^2 - 25$ **104.** $x^2 + 20x + 100$

105. $j^2 - k^4$ **106.** $9x^2 - 42x + 49$

107. $81x^2 + 144x + 64$ **108.** $16b^4 - 121c^6$

7-6 Choosing a Factoring Method

EXAMPLES

■ **Tell whether $(3x - 9)(x + 4)$ is completely factored. If not, factor it.**

$(3x - 9)(x + 4)$ — *$3x - 9$ can be factored.*

$3(x - 3)(x + 4)$ — *Factor out 3, the GCF of $3x$ and 9.*

■ $3ab^2 - 48a$

$3a(b^2 - 16)$ — *Factor out the GCF.*

$3a(b + 4)(b - 4)$ — *Factor the difference of two squares.*

Check $3a(b + 4)(b - 4) = 3a(b^2 - 16)$

$= 3ab^2 - 48a$ ✓

■ $2m^3 + 4m^2 - 48m$

$2m(m^2 + 2m - 24)$ — *Factor out the GCF.*

$2m(m - 4)(m + 6)$ — *Factor the trinomial.*

Check $2m(m - 4)(m + 6)$

$2m(m^2 + 2m - 24)$

$2m^3 + 4m^2 - 48m$ ✓

EXERCISES

Tell whether each polynomial is completely factored. If not, factor it.

109. $4x^2 + 10x + 6 = (4x + 6)(x + 1)$

110. $3y^2 + 75 = 3(y^2 + 25)$

111. $b^4 - 81 = (b^2 + 9)(b^2 - 9)$

112. $x^2 - 6x + 9 = (x - 3)^2$

Factor each polynomial completely. Check your answer.

113. $4x^2 - 64$ **114.** $3b^5 - 6b^4 - 24b^3$

115. $a^4b^3 - a^2b^5$ **116.** $t^{20} - t^4$

117. $5x^2 + 20x + 15$ **118.** $2x^4 - 50x^2$

119. $8t + 32 + 2st + 8s$

120. $25m^3 - 90m^2 - 40m$

121. $32x^4 - 48x^3 + 8x^2 - 12x$

122. $6s^4t + 12s^3t^2 + 6s^2t^3$

123. $10m^3 + 4m^2 - 90m - 36$

Answers

91. yes; $(x + 6)^2$

92. no; $5x \neq 2(x)(5)$

93. no; $-2x \neq 2(2x)(1)$

94. yes; $(3x + 2)^2$

95. no; $8x \neq 2(4x)(2)$

96. yes; $(x + 7)^2$

97. yes; $(10x - 9)(10x + 9)$

98. No; 2 is not a perfect square.

99. No; 5 and 10 are not perfect squares.

100. yes; $(-12 + x^3)(-12 - x^3)$

101. no; not a difference

102. yes; $(10p - 5q)(10p + 5q)$

103. $(x - 5)(x + 5)$; difference of 2 squares

104. $(x + 10)^2$; perfect-square trinomial

105. $(j - k^2)(j + k^2)$; difference of 2 squares

106. $(3x - 7)^2$; perfect-square trinomial

107. $(9x + 8)^2$; perfect-square trinomial

108. $(4b^2 - 11c^3)(4b^2 + 11c^3)$; difference of 2 squares

109. no; $2(2x + 3)(x + 1)$

110. yes

111. no; $(b^2 + 9)(b - 3)(b + 3)$

112. yes

113. $4(x - 4)(x + 4)$

114. $3b^3(b - 4)(b + 2)$

115. $a^2b^3(a - b)(a + b)$

116. $t^4(t^8 + 1)(t^4 + 1)(t^2 + 1)(t + 1)(t - 1)$

117. $5(x + 3)(x + 1)$

118. $2x^2(x - 5)(x + 5)$

119. $2(s + 4)(t + 4)$

120. $5m(5m + 2)(m - 4)$

121. $4x(4x^2 + 1)(2x - 3)$

122. $6s^2t(s + t)^2$

123. $2(m + 3)(m - 3)(5m + 2)$

CHAPTER 7 CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

9. $(3 + 4c)(c - 5)$
10. $(5x^2 + 2)(2x - 5)$
11. $(y - 1)(4y^2 + 3)$
29. $8x(x + 4)(x + 5)$

Learn It Online
State Resources Online

CHAPTER 7 CHAPTER TEST

Find the GCF of each pair of monomials.

1. $3t^4$ and $8t^2$ t^2
2. $2y^3$ and $-12y$ $2y$
3. $15n^5$ and $9n^4$ $3n^4$
4. Write the prime factorization of 360. $2^3 \cdot 3^2 \cdot 5$
5. A coin collector is arranging a display of three types of nickels. The types of nickels and number of each type are shown in the table. The collector wants to arrange them in rows with the same number in each row without having different types in the same row. How many rows will she need if she puts the greatest possible number of nickels in each row? 10 rows

Type of Nickel	Number of Nickels
Liberty	16
Buffalo	24
Jefferson	40

Factor each expression.

6. $24m^2 + 4m^3$ $4m^2(6 + m)$
7. $9x^5 - 12x$ $3x(3x^4 - 4)$
8. $-2r^4 - 6$ $-2(r^4 + 3)$
9. $3(c - 5) + 4c(c - 5)$
10. $10x^3 + 4x - 25x^2 - 10$
11. $4y^3 - 4y^2 - 3 + 3y$
12. A model rocket is shot vertically from a deck into the air at a speed of 50 m/s. The expression $-5t^2 + 50t + 5$ gives the approximate height of the rocket after t seconds. Factor this expression. $-5(t^2 - 10t - 1)$

Factor each trinomial.

13. $x^2 + 6x + 5$ $(x + 5)(x + 1)$
14. $x^2 - 4x - 21$ $(x - 7)(x + 3)$
15. $x^2 - 8x + 15$ $(x - 5)(x - 3)$
16. $2x^2 + 9x + 7$ $(2x + 7)(x + 1)$
17. $2x^2 + 9x - 18$ $(2x - 3)(x + 6)$
18. $-3x^2 - 2x + 8$ $-1(3x - 4)(x + 2)$

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

19. $a^2 + 14a + 49$ yes; $(a + 7)^2$
20. $2x^2 + 10x + 25$ No; $2x^2$ is not a perf. square.
21. $9t^2 - 6t + 1$ yes; $(3t - 1)^2$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

22. $b^2 - 16$ yes; $(b - 4)(b + 4)$
23. $25y^2 - 10$ No; 10 is not a perf. square.
24. $9a^2 - b^{10}$ yes; $(3a - b^5)(3a + b^5)$
25. A company is producing square sheets of plastic. Each has an area of $(9x^2 + 30x + 25)$ ft^2. The dimensions of each sheet are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of a sheet. Find the perimeter when $x = 4$ ft. $P = 4(3x + 5)$ ft; 68 ft

Tell whether each expression is completely factored. If not, factor it.

26. $(6x - 3)(x + 5)$ no; $3(2x - 1)(x + 5)$
27. $(v^5 + 10)(v^5 - 10)$ yes
28. $(2b + 3)(3b - 2)$ yes

Factor each polynomial completely.

29. $8x^3 + 72x^2 + 160x$ $8x(x + 4)(x + 5)$
30. $3x^5 - 27x^3$ $3x^3(x - 3)(x + 3)$
31. $8x^3 + 64x^2 - 20x - 160$ $4(2x^2 - 5)(x + 8)$
32. $cd^4 - c^7d^6$ $cd^4(1 - c^3d)(1 + c^3d)$
33. $100x^2 - 80x + 16$ $4(5x - 2)^2$
34. $7m^8 - 7$ $7(m^4 + 1)(m^2 + 1)(m + 1)(m - 1)$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 7

FOCUS ON ACT

The ACT Mathematics test booklet usually has writing space for scratch work. You may not bring your own scratch paper to the testing center. Remember that any scratch work done in the test booklet is for your use only and will not be scored. Be sure to transfer your final answer to the answer sheet.

HOT TIP! If you are unsure how to solve a problem, look through the answer choices. They may provide you with a clue to the solution method. It may take longer to work backward from the answer choices, so make sure you monitor your time.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. What is the value of $c^2 - d^2$ if $c + d = 7$ and $c - d = -2$?

(A) -14
(B) -5
(C) 5
(D) 14
(E) 45

2. Which of the following is the complete factorization of $6a^3b + 3a^2b^3$?

(F) $6a^3b^3$
(G) $9a^5b^4$
(H) $3ab(2a^2 + ab^2)$
(J) $3a^2b(2a + b^2)$
(K) $(6a^3b)(3a^2b^3)$

3. Which of the following is a factor of $x^2 + 3x - 18$?

(A) $x + 2$
(B) $x + 3$
(C) $x + 6$
(D) $x + 9$
(E) $x + 18$

4. The binomial $x - 3$ is NOT a factor of which of the following trinomials?

(F) $2x^2 - x - 3$
(G) $2x^2 - 5x - 3$
(H) $2x^2 - 8x + 6$
(J) $3x^2 - 6x - 9$
(K) $3x^2 - 10x + 3$

5. For what value of n is $4x^2 + 20x + n^2 = (2x + n)^2$ true for any real number x?

(A) 4
(B) 5
(C) 8
(D) 10
(E) 25

6. What is the factored form of $x^2 + \frac{2x}{3} + \frac{x}{2} + \frac{2}{6}$?

(F) $\left(x + \frac{1}{3}\right)\left(x + \frac{1}{2}\right)$
(G) $\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right)$
(H) $\left(x + \frac{2}{3}\right)\left(x + \frac{1}{6}\right)$
(J) $(x + 2)\left(x + \frac{1}{3}\right)$
(K) $\left(x + \frac{1}{3}\right)\left(x + \frac{2}{3}\right)$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 7

Organizer

Objective: Provide practice for college entrance exams such as the ACT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the ACT represent the following content areas:

Pre-Algebra, 23%
Elementary Algebra, 17%
Intermediate Algebra, 15%
Coordinate Geometry, 15%
Plane Geometry, 23%
Trigonometry, 7%

Items on this page focus on:

• Elementary Algebra

TEST PREP DOCTOR

1. Students who chose **B** may have added the values of the factors instead of multiplying them. Students who chose **E** squared the value of each factor and subtracted the results.

2. Students who chose **H** found a factored form of the expression but did not find the greatest common factor to obtain the complete factorization. Remind students to check for any remaining common factors once they think they are done factoring.

3. Students who chose **B** may have factored the polynomial with the correct numerical values but the incorrect signs, i.e., as $(x + 3)(x - 6)$.

4. Students may spend too much time on this item if they try to factor each answer choice. Suggest that students start by considering the form of the unknown factor, given the coefficient of the x^x-term in each choice, and then use the constant term of the trinomial to find a possible constant term of the unknown factor.

5. Students who chose **E** found the value of n^2 instead of n. Students who chose **D** may have mistakenly thought that the middle term is twice the product of the terms of the squared binomial.

6. Students who did not choose **G** should check their work by multiplying the factors and seeing whether their answer matches the given expression.

CHAPTER 7 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

INTERACTIVE **Online Edition**

TEST PREP DOCTOR This Test Tackler describes how to use key words and context clues to translate a word problem into a mathematical equation or expression. Encourage students to refer to the table as they answer the questions.

Any Question Type: Translate Words to Math

When reading a word problem, look for key words and context clues to help you translate the words into a mathematical equation or expression.

Some key words, such as those shown in this table, represent certain mathematical operations.

Action	Math Operation
Combining, increasing	Addition
Decreasing, reducing	Subtraction
Increasing or decreasing by a factor	Multiplication
Separating	Division

EXAMPLE

Short Response **The polynomial $x^2 + 7x + 12$ represents the area of a rectangle in square meters. The width is $(x + 3)$ meters. Find the combined measure of the length and the width.**

Use action words and context clues to translate the words into equations.

$x^2 + 7x + 12$ represents the area of a rectangle in square meters.
$x^2 + 7x + 12 = A$
The width is $(x + 3)$ meters.
$w = (x + 3)$
Find the combined measure of the length and the width.
$m = \ell + w$

Now use the equations to solve the problem.

$A = \ell w$ — *Write the formula for area of a rectangle.*

$x^2 + 7x + 12 = \ell(x + 3)$ — *Substitute $x^2 + 7x + 12$ for A and $(x + 3)$ for w.*

$(x + \ ?\ )(x + 3)$ — *Factor $x^2 + 7x + 12$ to find an expression for the length.*

$(x + 4)(x + 3)$ — *3(4) = 12; 3 + 4 = 7*

The length is $(x + 4)$.

$m = \ell + w$ — *Write the equation for the combined measure of the length and width.*

$m = (x + 4) + (x + 3)$ — *Substitute $(x + 4)$ for ℓ and $(x + 3)$ for w.*

$m = 2x + 7$ — *Combine like terms.*

The combined measure of the length and width is $(2x + 7)$ meters.

HOT TIP! Sometimes you cannot write an expression or equation in the order that the key words appear. For example, the expression "4 years younger than Maria" is written mathematically as $m - 4$.

Read each test item and answer the questions that follow.

Item A

Short Response The width of Alvin's rectangular mural is 6 times the length x. Alvin plans to make a new mural with an area of $(6x^2 - 24x + 24)$ square meters. By how much did Alvin decrease the area of the mural? Show your work.

1. What key words or context clues are in the first sentence of the test item? Use these clues to write an expression that represents the width of the rectangle.
2. Write an equation to represent the area of Alvin's first mural.
3. What math operation does the key word *decrease* represent?

Item B

Multiple Choice Which factored expression represents the phrase shown below?

the square of the number of hours it takes to empty a cistern minus 20 times the number of hours plus 64

Ⓐ $(h - 16)(h - 4)$ Ⓒ $(h - 8)(h - 8)$

Ⓑ $(h^2 - 20)(h - 64)$ Ⓓ $(h - 16)(h + 4)$

4. Which word in the phrase tells you to use an exponent in your expression?
5. What is the unknown value in the expression? Define a variable to represent this value.
6. Identify other key words and the mathematical operation phrase each one represents.

Item C

Multiple Choice A company owns two packaging plants. The polynomial $0.05x^2 + 16x - 9400$ models one plant's profit, where x is the number of units packaged. The polynomial $-0.01x^2 + 17x - 5400$ models the other plant's profit. If x is 25,000, what is the total profit of both plants?

Ⓐ −$5,830,300

Ⓑ $25,810,200

Ⓒ $31,640,500

Ⓓ $37,471,000

7. What mathematical symbol does the word *models* represent?
8. Write an equation for each plant that can be used to determine its profit P.
9. What mathematical operation does the term "total profit" represent?

Item D

Gridded Response One of the bases of a trapezoid is 12 meters greater than its height. The other base is 4 meters less than its height. Find the area of the trapezoid when the height is 6 meters.

10. Identify the unknown dimension, and assign it a variable.
11. A student is unsure how many bases a trapezoid has. Identify the context clues that can help this student.
12. Make a list of the key words in the problem, and link each word to its mathematical meaning.
13. Write an expression for each base of the trapezoid.

Answers

1. *is, times*; $w = 6x$
2. $A = 6x^2$
3. subtraction
4. *square*
5. the number of hours; h
6. *minus* represents subtraction; *plus* represents addition
7. equal sign
8. $P_1 = 0.05x^2 + 16x - 9400$; $P_2 = -0.01x^2 + 17x - 5400$
9. addition
10. height; h
11. "One of the bases"; "the other base"; these clues show that there are 2 bases.
12. *Is* means "equals." *Greater than* represents addition. *Less than* represents subtraction.
13. $b_1 = h + 12$; $b_2 = h - 4$

Answers to Test Items

A. $(24x - 24)$ m^2

B. A

C. B

D. 60 m^2

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources

Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

Learn It Online
State Test Practice

CUMULATIVE ASSESSMENT

Multiple Choice

1. A rectangle has an area of $(x^2 + 5x - 24)$ square units. Which of the following are possible expressions for the length and the width of the rectangle?

Ⓐ Length: $(x - 24)$ units; width: $(x + 1)$ units

Ⓑ Length: $(x - 4)$ units; width: $(x + 6)$ units

Ⓒ Length: $(x - 3)$ units; width: $(x + 8)$ units

Ⓓ Length: $(x + 12)$ units; width: $(x - 2)$ units

2. Which property of real numbers is used to transform the equation in Step 1 into the equation in Step 2?

Step 1: $4(x - 5) + 8 = 88$
Step 2: $4x - 20 + 8 = 88$
Step 3: $4x - 12 = 88$
Step 4: $4x = 100$
Step 5: $4x = 25$

Ⓕ Commutative Property of Multiplication

Ⓖ Associative Property of Multiplication

Ⓗ Multiplication Property

Ⓙ Distributive Property

3. If $\frac{2}{3}x - 9 = 3$, what is the value of the expression $8x - 3$?

Ⓐ -75 Ⓒ 61

Ⓑ -35 Ⓓ 141

4. Carlos and Bonita were just hired at a manufacturing plant. Carlos will earn $12.50 per hour. He will receive a hiring bonus of $300. Bonita will not get a hiring bonus, but she will earn $14.50 per hour. Which equation can you use to determine the number of hours h when both employees will have earned the same total amount?

Ⓕ $300 + 14.50h = 12.50h$

Ⓖ $14.50h + 300 = 12.50h$

Ⓗ $14.50h + 12.50h = 300$

Ⓙ $300 + 12.50h = 14.50h$

5. Which of the following expressions is equivalent to $x^2 - 8x + 16$?

Ⓐ $(x + 4)^2$ Ⓒ $(x + 8)(x + 2)$

Ⓑ $(x + 4)(x - 4)$ Ⓓ $(x - 4)^2$

6. An amusement park has two jumping attractions called moonbounces in the shape of similar cubes. The larger moonbounce has a volume of 4800 cubic feet. The smaller moonbounce is half the length of the larger one. What is the volume of the smaller moonbounce?

Ⓕ 300 cubic feet

Ⓖ 600 cubic feet

Ⓗ 1200 cubic feet

Ⓙ 2400 cubic feet

7. What is the value of y if the line through $(1, -1)$ and $(2, 2)$ is parallel to the line through $(-2, 1)$ and $(-1, y)$?

Ⓐ -8 Ⓒ 3

Ⓑ -2 Ⓓ 4

8. Which of the following shows the complete factorization of $2x^3 + 4x^2 - 6x$?

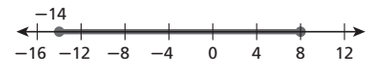
Ⓕ $(2x^2 - 2x)(x + 3)$

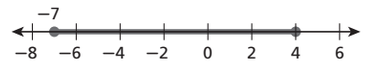
Ⓖ $2x(x^2 + 2x - 3)$

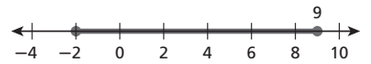
Ⓗ $2x(x - 1)(x + 3)$

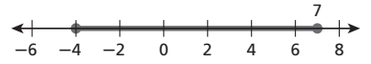
Ⓙ $2(x^3 + 2x^2 - 3x)$

9. Which graph shows the solution set of the compound inequality $-9 \leq 5 - 2x \leq 13$?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Answers

1. C
2. J
3. D
4. J
5. D
6. G
7. D
8. H
9. D
10. F
11. D
12. G
13. C
14. 3
15. $\frac{1}{4}$
16. 29.40
17. 10

HOT TIP! **Many standardized test booklets have a page listing commonly used formulas and basic measurement facts. Before the test begins, ask if you can remove the page from the booklet. If allowed, place the page nearby for easy reference.**

10. Which point lies on the graph of both functions?

$f(x) = 2x - 10$
$g(x) = 10 - 2x$

Ⓕ $(5, 0)$ Ⓗ $(0, 0)$

Ⓖ $(1, -8)$ Ⓙ $(2, 6)$

11. Hayley plans to solve the system of equations below.

$$\begin{cases} x + 3y = 8 \\ 5x - y = 8 \end{cases}$$

Which of the following does NOT show an equation Hayley can use to solve the system of equations?

Ⓐ $x + 3(5x - 8) = 8$

Ⓑ $5(8 - 3y) - y = 8$

Ⓒ $x = 8 - 3y$

Ⓓ $5x - (-x + 8) = 8$

12. Which value of b would make $x^2 + bx - 2$ factorable?

Ⓕ -2 Ⓗ 0

Ⓖ -1 Ⓙ 3

13. Which expression is equivalent to $5(x - 2) + 4x$?

Ⓐ $5x - 10$ Ⓒ $9x - 10$

Ⓑ $9x - 2$ Ⓓ $20x - 2$

Gridded Response

14. The complete factorization of $-12x^3 + 14x^2 + 6x$ is $-2x(ax + 1)(2x - 3)$. What is the value of a?

15. The expression $x^2 + x + b$ is a perfect-square trinomial. What is the value of b?

16. Margaret is buying a $35 sweater that is on sale for 20% off. What is the total price of the sweater, in dollars, when 5% sales tax is added?

17. A car is traveling at a constant speed. In 4 hours, the car travels 220 miles. How many hours will it take the car to travel 550 miles?

Short Response

18. The area of a certain circle is $\pi(9x^2 + 6x + 1)$ square centimeters. Find an expression for the length of the circle's radius. Explain how you found your answer.

19. A rectangle has an area of $(x^2 - 25)$ square feet.

a. Use factoring to write possible expressions for the length and width of the rectangle.

b. Use your expressions from part **a** to write an expression for the perimeter of the rectangle. Simplify the expression.

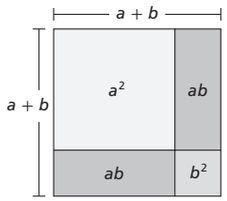
c. Use your expressions from parts **a** and **b** to find the perimeter and the area of the rectangle when $x = 10$ feet. Show your work.

20. What are 2 values of b that will make $2x^2 - bx - 20$ factorable? Explain your answer.

21. Show that you can factor the expression $x^2y - 12 + 3y - 4x^2$ by grouping in two different ways.

Extended Response

22. The diagram below can be used to show that the expression $(a + b)^2$ is equivalent to the expression $a^2 + 2ab + b^2$.



a. Make a diagram similar to the one above to model the expression $(a + b + c)^2$. Label each distinct area.

b. Use the labels from your diagram to write an expression equivalent to $(a + b + c)^2$.

c. Show that your expression in part **b** is equivalent to $(a + b + c)^2$ by evaluating each expression for $a = 4$, $b = 2$, and $c = 1$.

d. Factor $x^2 + y^2 + 9 + 2xy + 6x + 6y$. Show or explain how you found your answer.

Short-Response Rubric

Items 18–21

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 22

4 Points = The student correctly draws and labels a diagram in part **a,** writes a correct expression in part **b,** correctly evaluates the expression in part **c,** and correctly factors in part **d** with an explanation.

3 Points = The student correctly draws and labels **a** diagram in part a and writes a correct expression in part **b** but does not show any work or give an explanation for part **c** or part **d.**

2 Points = The student answers all parts correctly but does not show any work or explanation, or the student correctly answers parts **a** and **b** but does not correctly answer parts **c** and **d.**

1 Point = The student draws and labels a diagram in part **a** but does not correctly answer other parts of the problem, or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly or does not attempt all parts of the problem.

Answers

18. $A = \pi r^2 = (9x^2 + 6x + 1)\pi = (3x + 1)^2\pi$, so $r = (3x + 1)$ cm

19a. $\ell = (x + 5)$ ft; $w = (x - 5)$ ft

b. $P = 4x$ ft

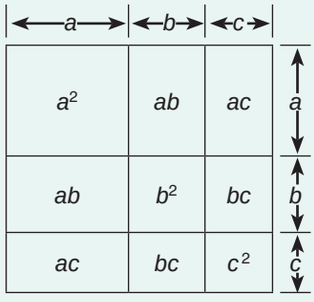
c. $P = 40$ ft; $A = 75$ ft^2

20. Any 2 of –39, –6, –3, 3, 6, 39; check students' explanations.

21. (1) $x^2y - 12 + 3y - 4x^2 =$
$(x^2y - 4x^2) + (3y - 12) =$
$x^2(y - 4) + 3(y - 4) =$
$(x^2 + 3)(y - 4)$

(2) $x^2y - 12 + 3y - 4x^2 =$
$(x^2y + 3y) + (-4x^2 - 12) =$
$y(x^2 + 3) - 4(x^2 + 3) =$
$(y - 4)(x^2 + 3)$

22a.



b. $a^2 + 2ab + b^2 + 2ac + 2bc + c^2$

c. Both expressions should equal 49.

d. $(x + y + 3)^2$; check students' work.

Real-World CONNECTIONS

Organizer

Objective: Choose appropriate problem-solving strategies and use them with skills from this chapter to solve real-world problems.

Online Edition

Alligator Adventure

Reading Strategies

In **Problem 2,** ask students what other question could be inserted before the actual question to make the problem easier. Possible answer: How many baby alligators would be in the nest?

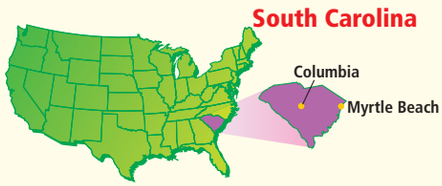
Problem-Solving Focus

In **Problem 3,** students should recognize that $y = 3x - 4$ is a linear function and that its graph forms a line. Therefore, they need to use only the Guess and Test strategy with the other two functions. Students may use the Find a Pattern strategy once they notice that the graphs are symmetric about the y-axis.

Discuss with students the importance of substituting both positive and negative values for x because a portion of $y = |x^2 - 4|$ may appear to be linear.

Learn It Online
State Resources Online

Real-World CONNECTIONS



Alligator Adventure

Alligator Adventure is a tourist attraction in North Myrtle Beach, South Carolina. Its exhibits feature alligators, snakes, Galapagos tortoises, bears, frogs, lizards, tropical birds, and crocodiles.

Choose one or more strategies to solve each problem.

1. At birth, alligators are about 8 inches long. Adult male alligators can grow up to 18 feet long. How many times as long as a newborn alligator is an alligator that measures 18 feet? **27 times**

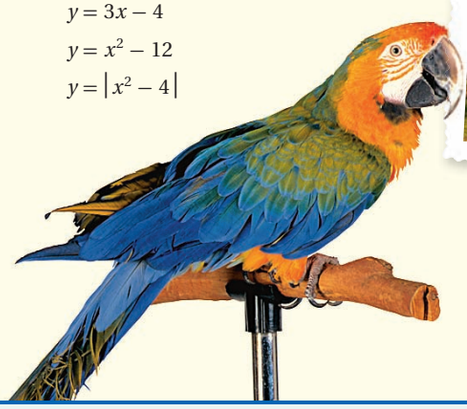
2. Alligators build their nests out of dirt, grass, and leaves. The female alligator will deposit 40 to 50 eggs in her nest. Suppose there were 40 eggs in a nest and 80 percent of them hatched. The baby alligators each weigh 3 ounces. What is the total weight of the hatched alligators? **96 oz**

3. Most alligators have U-shaped snouts while most crocodiles have V-shaped snouts. Which of the equations below has a graph that could model the shape of an alligator's snout? Explain.

   $y = 3x - 4$

   $y = x^2 - 12$

   $y = |x^2 - 4|$



3. $y = x^2 - 12$**; when several points generated by this equation are graphed, they form a U shape.**

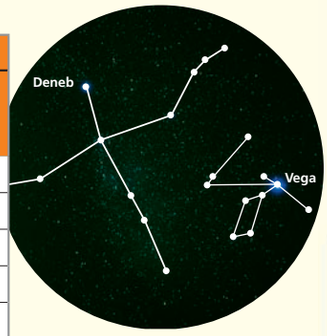
(tr), Eric Horan; (br), Eric Horan; (bl), PhotoDisc/Getty Images

Melton Memorial Observatory

Melton Memorial Observatory is run by the University of South Carolina in Columbia, South Carolina. The table lists some of the stars that can be seen from the observatory.

A light-year is the distance light can travel in one year, which is 9,500,000,000,000 kilometers.

Eight of the Brightest Stars as Seen from Earth				
Name	Rank	Apparent Magnitude	Distance from Earth (light-years)	Temperature (kelvins)
Sirius	1	−1.46	8.6	9,400
Arcturus	4	−0.04	34	4,290
Vega	5	0.03	25	9,600
Procyon	8	0.38	11.4	6,500
Altair	12	0.77	16	7,550
Spica	14	0.98	220	22,400
Deneb	20	1.25	1500	8,400
Regulus	25	1.35	69	12,000



Choose one or more strategies and use the table to solve each problem.

1. Order the stars from closest to Earth to farthest from Earth.

2. Identify which star is farthest from and which is closest to Earth. Approximate how many times as far as the closest star the farthest star is.

Deneb is farthest, and Sirius is closest; about 175 times

Apparent magnitude is a measure of how bright a star (or other object) appears from Earth. The lower the apparent magnitude of a star, the brighter the star appears.

3. Describe the correlation between apparent magnitude and distance from Earth in light-years. Make a scatter plot to support your answer.

A star's distance from Earth does not determine its apparent magnitude.

(tl), Courtesy Mike Brown/University of South Carolina; (br), Mike Brown/University of South Carolina, University Publications; (cr), John Sanford/Science Photo Library/Photo Researchers, Inc.

Melton Memorial Observatory

Reading Strategies

ENGLISH LANGUAGE LEARNERS

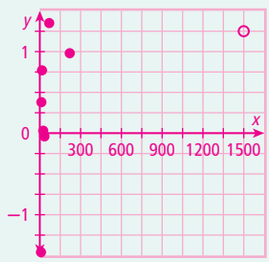
Tell students that in everyday life, *magnitude* refers to the extent or importance of something. For instance, if someone is describing the *magnitude of a problem,* they are discussing how serious it is.

Using Data Ask students to compare the second and third columns of the table. What do they notice? The magnitudes get larger as the ranks increase. Have students find the median magnitude. 0.575

Problem-Solving Focus

Encourage students to use the four-step problem-solving process for **Problem 3.** Focus on the second step: **(2) Make a Plan.** Have students determine a reasonable scale and interval for each axis. Possible answer: magnitude: −1.5 to 1.5 by 0.25, distance: 0 to 1650 by 150

Answers

1. Sirius, Procyon, Altair, Vega, Arcturus, Regulus, Spica, Deneb

3.